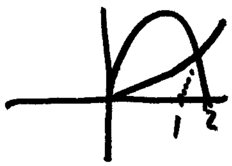
Exam 1B

MATH 230, FALL 2015

Name:

Directions: Show ALL work for full credit. No calculators allowed.

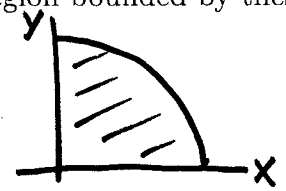
1. Find the area of the region bounded between the curves $y = x^3$ and $y = 2x - x^2$.



$$\int_0^1 (2x-x^2) - x^3\,dx = \int_0^1 -x^3 - x^2 + 2x\,dx = -\frac{1}{4}x^4 - \frac{1}{3}x^3 + x^2 \Big|_{x=0}^{x=1} = \frac{5}{12}$$

2. Use the curves $y = 4 - x^2$, $x = 0$, and $y = 0$ and $x \geq 0$ to answer the following questions.

a) Graph the region bounded by these curves.



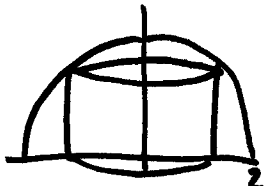
b) SET UP (do not evaluate) an integral representing the volume of the solid created when rotating the region about the given lines and using the given methods:

i) y-axis using disks/washers

IN TERMS OF Y: $y = 4 - x^2 \rightarrow x = \sqrt{4-y}$

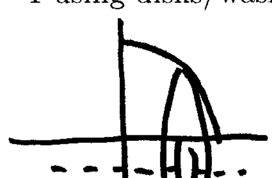
$$\int_0^4 \pi\big(\overset{R(y)}{\sqrt{4-y}}\big)^2\,dy$$

ii) y-axis using cylindrical shells



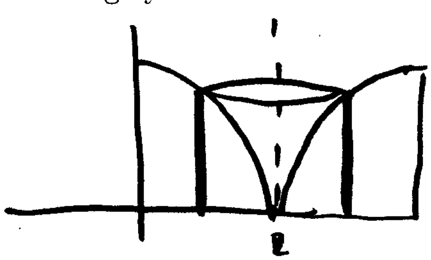
$$\int_0^2 2\pi \overset{R}{x}\,\overset{H}{(4-x^2)}\,dx$$

iii) $y = -1$ using disks/washers



$$\int_0^2 \pi\big(\overset{R(x)}{4-x^2+1}\big)^2 - \pi\big(\overset{r(x)}{1}\big)^2\,dx = \int_0^2 \pi(5-x^2)^2 - \pi\,dx$$

iv) $x = 2$ using cylindrical shells



$$\int_0^2 2\pi \overset{R}{(2-x)}\,\overset{H}{(4-x^2)}\,dx$$

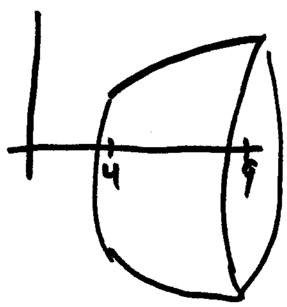
3. Set up an integral that represents the length of the curve $y = xe^{-x}$ for $0 \le x \le 2$.

$$L = \int_0^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

(1) $\frac{dy}{dx} = xe^{-x}\cdot(-1) + (1)e^{-x} = -xe^{-x} + e^{x}$

(2) $L = \int_0^2 \sqrt{1+\left(-xe^{-x}+e^{x}\right)^2}\,dx$

4. Find the exact area of the surface obtained by rotating the curve $y = \sqrt{x}$ between $4 \le x \le 9$ about the x-axis.



$$SA = \int_4^9 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_4^9 2\pi\sqrt{x}\sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}\,dx$$

ON BACK

$$= \int_4^9 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx = \int_4^9 2\pi\sqrt{x+\frac{1}{4}}\,dx$$

5. Find $(f^{-1})'(1)$ for $f(x) = 6x^3 + 2x + 1$ and describe what this value represents.

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = \frac{1}{18(0)^2+2} = \frac{1}{2}$$

(1) $f^{-1}(1) = x$ WHERE $f(x) = 1$

$6x^3 + 2x + 1 = 1$

$x = 0$

$f^{-1}(1) = 0$

(2) $f'(x) = 18x^2 + 2$

6. Use logarithmic differentiation to differentiate $y = (\cos(x))^x$.

LOG BOTH SIDES: $\ln y = x \ln \cos x$

DIFFERENTIATE: $\frac{d}{dx}[\ln y] = \frac{d}{dx}[x \ln \cos x]$

$\Rightarrow \frac{1}{y}\cdot y' = x\cdot\frac{1}{\cos x}\cdot -\sin x + (1)\ln\cos x$

$y' = y(-x\tan x + \ln\cos x)$

$y' = (\cos x)^x(-x\tan x + \ln\cos x)$

(1) LET $u = x + \frac{1}{4}$

(2) $du = dx$

(3) IF $x = 9$, $u = 9.25$

IF $x = 4$, $u = 4.25$

(3) SUBSTITUTE

$$\int_4^9 2\pi \sqrt{x+\tfrac{1}{4}}\,dx = \int_{4.25}^{9.25} 2\pi \sqrt{u}\,du$$

$$= \int_{4.25}^{9.25} 2\pi u^{1/2}\,du$$

$$= \frac{4}{3}\pi u^{3/2} \Big|_{u=4.25}^{u=9.25}$$

$$= \frac{4}{3}\pi (9.25)^{3/2} - \frac{4}{3}\pi (4.25)^{3/2}$$

7. Evaluate the integral.

a) $\displaystyle\int_1^e \frac{x^2+x+1}{x}\,dx$

$$= \int_1^e \frac{x^2}{x} + \frac{x}{x} + \frac{1}{x}\,dx$$

$$= \int_1^e x + 1 + \frac{1}{x}\,dx = \frac{1}{2}x^2 + x + \ln|x| \Big|_{x=1}^{x=e} = \left[\left(\frac{1}{2}e^2 + e + \ln e\right) - \left(\frac{1}{2}(1)^2 + 1 + \ln 1\right)\right]$$

$$= \frac{1}{2}e^2 + e - \frac{1}{2}$$

b) $\displaystyle\int \frac{\sin(\ln(x))}{x}\,dx$

(1) LET $u = \ln x$

(2) $du = \frac{1}{x}\,dx$

(3) SUBSTITUTE $\nearrow$ $\displaystyle\int \sin u\,du = -\cos u + C$

$$= -\cos(\ln x) + C$$

c) $\displaystyle\int e^x(4+e^x)^5\,dx$

(1) LET $u = 4 + e^x$

(2) $du = e^x\,dx$

(3) SUBSTITUTE $\nearrow$ $\displaystyle\int u^5\,du = \frac{1}{6}u^6 + C$

$$= \frac{1}{6}(4+e^x)^6 + C$$

Extra Credit Evaluate $\displaystyle\int \tan(x)\,dx. = \int \frac{\sin x}{\cos x}\,dx$

(1) LET $u = \cos x$

(2) $du = -\sin x\,dx$

$\Rightarrow -du = \sin x\,dx$

(3) SUBSTITUTE $\nearrow$ $\displaystyle\int \frac{-1}{u}\,du = -\ln|u| + C$

$$= -\ln|\cos x| + C$$

OR $\ln|\sec x| + C$