EXAM 3 MATH 230, CALCULUS II NAME: Solution

Show all work to receive full credit.

1. In each case, answer CONVERGES, DIVERGES, or INCONCLUSIVE.

   (a) If a_n are nonnegative, decreasing to 0, then $\sum a_n$ ... Inconclusive

   (b) If $0 \leq a_n \leq \frac{1}{n}$, then $\sum a_n$ ... Inconclusive

   (c) If $0 \leq a_n \leq \frac{1}{n^3}$, then $\sum a_n$ ... Converges

   (d) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$, then $\sum a_n$ ... Diverges

2. State whether the following series converges or diverges. You do not need to show work.

   (a) $\sum \frac{n^2}{2^n}$ Converges Ratio Test

   (b) $\sum \frac{\ln n}{n}$ Diverges Integral Test

   (c) $\sum \frac{n!}{100^n}$ Diverges Ratio Test

   (d) $\sum \frac{(-1)^n}{n^2+1}$ Converges Alternating Series Test

   (e) $\sum \frac{(-1)^n}{\sqrt{n^2+1}}$ Converges Alternating Series Test

   (f) $\sum_{n=1}^{\infty} \cos(2n\pi) = \sum_{n=1}^{\infty} 1$ Diverges by the Divergence Test

$\lim_{n\to\infty} 1 \neq 0$

3. Sum the series $\sum \frac{4+(-3)^n}{5^{n+1}}$ or show that it diverges. Display your work carefully and completely. $\uparrow n=0$

$$\sum_{0} \frac{4}{5^{n+1}} + \sum_{0} \frac{(-3)^n}{5^{n+1}}$$

$$(1)\ \sum_{n=0} \frac{4}{5^{n+1}} = \sum_{0} \frac{4}{5\cdot 5^n} = \sum_{n=0} \frac{4}{5}\left(\frac{1}{5}\right)^n = \frac{4/5}{1-1/5} = 1$$

$$(2)\ \sum_{n=0} \frac{(-3)^n}{5^{n+1}} = \sum_{0} \frac{(-3)^n}{5\cdot 5^n} = \sum \frac{1}{5}\left(\frac{-3}{5}\right)^n = \frac{1/5}{1-(-3/5)} = \frac{1/5}{8/5} = \frac{1}{8}$$

Final: $1 + \frac{1}{8} = \underline{\underline{\frac{9}{8}}}$

4. Compute $\int_2^\infty \frac{2}{x(\ln x)^3}\,dx$.

Let $u = \ln x$, $du = \frac{+1}{x}dx$. If $x=\infty$, $u=\infty$,
$x=2$, $u=\ln 2$

$$\int_{\ln 2}^{\infty} \frac{2}{u^3}\,du = \lim_{b\to\infty} \int_{\ln 2}^{b} 2u^{-3}\,du = \lim_{b\to\infty} \left[-u^{-2}\Big|_{\ln 2}^{b} \right]$$

$$= \lim_{b\to\infty} \left[-\frac{1}{b^2} + \frac{1}{(\ln 2)^2} \right] = 0 + \frac{1}{(\ln 2)^2}.$$

$$\text{So } \int_2^\infty \frac{2}{x(\ln x)^3}\,dx = \frac{1}{(\ln 2)^2}$$

5. Determine whether each series diverges, converges absolutely, or converges conditionally. Support your answer with a clear argument based on our tests.

(a) $\sum \frac{n^2(-2)^n}{n!}$. Ratio Test. $a_n = \frac{n^2(-2)^n}{n!}$, $a_{n+1} = \frac{(n+1)^2(-2)^{n+1}}{(n+1)!}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^2(-2)^{n+1}}{(n+1)!}\cdot\frac{n!}{n^2(-2)^n}\right| = \lim\left|\frac{(n+1)^2(-2)^n\cdot(-2)}{(n+1)\cdot n!}\cdot\frac{n!}{n^2(-2)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)^2}{n^2}\cdot\frac{(-2)^n(-2)}{(-2)^n}\cdot\frac{n!}{(n+1)n!}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^2}{n^2}\cdot(-2)\cdot\frac{1}{n+1}\right|$$

$$= 1\cdot 2\cdot 0 = 0 < 1$$

Converges absolutely by Ratio Test

(b) $\sum \frac{(-1)^n}{3n+1}$. Let $b_n = \frac{1}{3n+1}$

• Since $\frac{1}{3n+1}$ is positive, decreasing, and $\lim \frac{1}{3n+1} = 0$, then $\sum\frac{(-1)^n}{3n+1}$ converges by AST. Now check for Absolute

• $\sum\left|\frac{(-1)^n}{3n+1}\right| = \sum\frac{1}{3n+1}$. Do a LCT with $\sum\frac{1}{n}$ (diverges by p-test)

$$\lim_{n\to\infty}\frac{\frac{1}{3n+1}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n}{3n+1} = \frac{1}{3} \text{ (finite \#)}$$

therefore $\sum\left|\frac{(-1)^n}{3n+1}\right|$ diverges. leaving $\sum\frac{(-1)^n}{3n+1}$ to ~~c~~ converge conditionally.

(c) $\sum\left(\frac{-3}{n}\right)^n$

Root Test

$$\lim_{n\to\infty}\sqrt[n]{\left|\left(-\frac{3}{n}\right)^n\right|} = \lim_{n\to\infty}\left|\frac{-3}{n}\right| = \underset{L}{0} < 1.$$ Since $L < 1$,

$\sum\left(\frac{-3}{n}\right)^n$ converges absolutely by the Root Test