EXAM 3 MATH 230, CALCULUS II NAME:

Show all work to receive full credit.

1. Suppose that $0 \leq a_n \leq b_n$ for all n. Mark each of the following statements clearly as TRUE or FALSE. If the answer is FALSE, given an example showing why.

   (a) If $\sum a_n$ diverges, then $\sum b_n$ must diverge.

   (b) If $\sum a_n$ diverges, then $\sum a_n^2$ must diverge.

   (c) If $\sum b_n$ converges, then $\sum (a_n + b_n)$ must converge.

   (d) If $\sum a_n$ diverges, then $\sum (-1)^n a_n$ must diverge.

   (e) If $\sum a_n$ converges, then $\sum (3 + \cos n) a_n$ must converge.

   (f) If $a_n \to 0$ then $\sum a_n$ must converge.

2. Consider the series

$$\sum_{n=1}^{\infty} a_n = 5 - \frac{5}{4} + \frac{5}{9} - \frac{5}{16} + ...$$

(a) Find a formula for a_n.

(b) Show that the series converges.

(c) Find a value of n such that the nth partial sum S_n approximates the sum of the series to within 0.001.

3. Evaluate $\displaystyle\int_2^\infty \frac{x}{(1+x^2)^2}\,dx$

4. Does $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx$ converge or diverge? Justify your answers.

5. Determine whether each of the following series diverges, converges conditionally, or converges absolutely. Justify your answers with tests.

(a) $\sum \frac{\sqrt{n^4+1}}{5n^2-4}$

(b) $\sum \frac{(-1)^n \ln n}{n}$

(c) $\sum \frac{n^5 4^{n+2}}{(-3)^n}$