Math 230 Calculus II
Practice problems for Exam II

Exam II will be based on Sections 6.3*, 6.4*, 6.6, 6.8, 7.1, 7.2, 7.3, and 7.4.

1. Review all definitions from Sections 6.3*, 6.4*, 6.6, 6.8, 7.1, 7.2, 7.3, and 7.4.

2. Review all theorems from Sections 6.3*, 6.4*, 6.6, 6.8, 7.1, 7.2, 7.3, and 7.4.

3. Solve each equation for x.

   (a) $e^{3x+1} = k$

   (b) $e^{2x} - e^x - 6 = 0$

   (c) $\ln(\ln x) = 1$

   (d) $\ln(2x + 1) = 2 - \ln x$

   Answer: (a) $\frac{1}{3}(\ln k - 1)$; (b) $\ln 3$; (c) e^e; (d) $\frac{-1+\sqrt{1+8e^2}}{4}$.

4. Find the limit.

   (a) $\displaystyle \lim_{x \to (\pi/2)^+} e^{\tan x}$

   (b) $\displaystyle \lim_{x \to 2^-} e^{3/(2-x)}$

   Answer: (a) 0; (b) ∞.

5. Differentiate the function.

   (a) $f(x) = \sqrt{1 + 2e^{3x}}$

   (b) $g(x) = e^{e^x}$

   (c) $h(x) = \sqrt{1 + xe^{-2x}}$

   (d) $k(t) = \sin(e^t) + e^{\sin t}$.

   Answer: (a) $\frac{3e^{3x}}{\sqrt{1+2e^{3x}}}$; (b) $e^{e^x + x}$; (c) $\frac{e^{-2x}(-2x+1)}{2\sqrt{1+xe^{-2x}}}$; (d) $e^t \cos(e^t) + e^{\sin t} \cos t$.

6. Evaluate the integral

   (a) $\displaystyle \int_{-5}^{5} e \, dx$

   (b) $\displaystyle \int e^{\tan x} \sec^2 x \, dx$

   (c) $\displaystyle \int e^x (4 + e^x)^5 \, dx$

   Answer: (a) $10e$; (b) $e^{\tan x} + C$; (c) $\frac{1}{6}(4 + e^x)^6 + C$.

7. Differentiate the function.

   (a) $f(x) = (\sqrt{x})^x$

   (b) $g(x) = (\ln x)^{\cos x}$

   (c) $h(x) = x^5 + 5^x$

   (d) $k(x) = \log_5(xe^x)$

   Answer: (a) $\frac{1}{2}(\sqrt{x})^x(1 + \ln x)$; (b) $(\ln x)^{\cos x}\left(\frac{\cos x}{x \ln x} - \sin x \ln \ln x\right)$; (c) $5x^4 + 5^x \ln 5$; (d) $\frac{x+1}{x \ln 5}$.

8. Find the inverse function of $f(x) = \log_{10}\left(1 + \dfrac{1}{x}\right)$. *Answer:* $\frac{1}{10^x - 1}$.

9. Find the exact value of each expression.

   (a) $\sin^{-1}\left(\dfrac{1}{2}\right)$

   (b) $\sin^{-1}(\sin(\pi/3))$

   (c) $\sin^{-1}(\sin(7\pi/3))$

   (d) $\tan\left(\sin^{-1}\left(\dfrac{2}{3}\right)\right)$

   Answer: (a) $\pi/6$; (b) $\pi/3$; (c) $\pi/3$; (d) $2/\sqrt{5}$.

10. Prove that $\cos\left(\sin^{-1}x\right) = \sqrt{1-x^2}$.

11. Simplify the expression $\sin\left(\tan^{-1}x\right)$. *Answer:* $\dfrac{x}{\sqrt{1+x^2}}$.

12. Differentiate the function.

    (a) $f(x) = x\sin^{-1}x + \sqrt{1-x^2}$

    (b) $g(x) = \cos^{-1}(e^{2x})$

    (c) $h(\theta) = \tan^{-1}(\cos\theta)$

    Answer: (a) $\sin^{-1}x$; (b) $-\dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$; (c) $-\dfrac{\sin\theta}{1+\cos^2\theta}$.

13. Evaluate the integral.

    (a) $\displaystyle\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

    (b) $\displaystyle\int \frac{1+x}{1+x^2}\,dx$

    (c) $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

    Answer: (a) $\pi^2/72$; (b) $\tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$; (c) $\pi/4$.

14. Find the limit.

    (a) $\displaystyle\lim_{x\to(\pi/2)^+} \frac{\cos x}{1-\sin x}$

    (b) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

    (c) $\displaystyle\lim_{x\to\pi/2} \frac{1-\sin x}{\csc x}$

    (d) $\displaystyle\lim_{x\to\infty} \frac{(\ln x)^2}{x}$

    (e) $\displaystyle\lim_{x\to\infty} \sqrt{x}e^{-x/2}$

    (f) $\displaystyle\lim_{x\to1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

    (g) $\displaystyle\lim_{x\to0}(1-2x)^{1/x}$

    (h) $\displaystyle\lim_{x\to\infty} x^{1/x}$

    Answer: (a) $-\infty$; (b) 0; (c) 0; (d) 0; (e) 0; (f) $1/2$; (g) $1/e^2$; (h) 1.

15. Evaluate the integral.

    (a) $\displaystyle\int te^{-3t}\,dt$

    (b) $\displaystyle\int x^5\ln x\,dx$

    Answer: (a) $-\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C$; (b) $\frac{1}{6}x^6\ln x - \frac{1}{36}x^6 + C$.

16. Evaluate the integral.

(a) $\displaystyle\int_0^{\pi/2} \sin^7\theta \cos^5\theta\, d\theta$. *Answer:* 1/120.

(b) $\displaystyle\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\, dx$. *Answer:* $\frac{2}{3}\cos^3(\sqrt{x}) - 2\cos\sqrt{x} + C$.

(c) $\displaystyle\int t\sin^2 t\, dt$. *Answer:* $\frac{1}{4}t^2 - \frac{1}{4}t\sin 2t - \frac{1}{8}\cos 2t + C$.

(d) $\displaystyle\int \tan x \sec^3 x\, dx$. *Answer:* $\frac{1}{3}\sec^3 x + C$.

(e) $\displaystyle\int \tan^2\theta \sec^4\theta\, d\theta$. *Answer:* $\frac{1}{5}\tan^5\theta + \frac{1}{3}\tan^3\theta + C$.

(f) $\displaystyle\int \frac{1 - \tan^2 x}{\sec^2 x}\, dx$. *Answer:* $\frac{1}{2}\sin 2x + C$.

17. Evaluate the integral

(a) $\displaystyle\int_0^1 x^3\sqrt{1 - x^2}\, dx$. *Answer:* 2/15.

(b) $\displaystyle\int_{\sqrt{2}}^2 \frac{1}{t^3\sqrt{t^2 - 1}}\, dt$. *Answer:* $\frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$.

(c) $\displaystyle\int \frac{1}{\sqrt{x^2 + 16}}\, dx$. *Answer:* $\ln(\sqrt{x^2 + 16} + x) + C$.

(d) $\displaystyle\int \sqrt{x^2 + 2x}\, dx$. *Answer:* $\frac{1}{2}(x + 1)\sqrt{x^2 + 2x} - \frac{1}{2}\ln|x + 1 + \sqrt{x^2 + 2x}| + C$.

(e) $\displaystyle\int x\sqrt{1 - x^4}\, dx$. *Answer:* $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1 - x^4} + C$.

18. Evaluate the integral.

(a) $\displaystyle\int \frac{5x + 1}{(2x + 1)(x - 1)}\, dx$. *Answer:* $\frac{1}{2}\ln|2x + 1| + 2\ln|x - 1| + C$.

(b) $\displaystyle\int_3^4 \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2}\, dx$. *Answer:* $\frac{7}{6} + \ln\frac{2}{3}$.

(c) $\displaystyle\int_0^1 \frac{x^3 - 4x - 10}{x^2 - x - 6}\, dx$. *Answer:* $\frac{3}{2} + \ln\frac{3}{2}$

(d) $\displaystyle\int \frac{x^2 - x - 6}{x^3 + 3x}\, dx$. *Answer:* $2\ln|x| - \frac{1}{2}\ln(x^2 + 3) - \frac{1}{\sqrt{3}}\tan^{-1}(\frac{x}{\sqrt{3}}) + C$.