EXAM 3 REVIEW MATH 230, CALCULUS II, FALL 2015 NAME:

Show all work to receive full credit.

1. Make sure you have a copy of the series convergence tests. It's in your best interest to know everything on that sheet.

   http://www.brianveitch.com/calculus2/lectures/series-strategy.pdf

2. Improper Integrals. Determine if the integral is convergent or divergent. There are two types but since it will be paired with the integral test you'll be using this: $\int_a^\infty f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$

3. Sequences: a_n

   (a) What is a sequence?

   (b) Does the sequence converge, i.e., $\lim_{n\to\infty} a_n = L$?

   (c) Make sure you know L'Hospitals Rule because you'll need it when evaluating limits.

   (d) Geometric Sequence: $ar^n = \{a, ar, ar^2, ar^3, ...\}$, where $\lim_{n\to\infty} r^n = 0$ when $-1 < r < 1$

   (e) Given a sequence $\{a_1, a_2, a_3, ...\}$ know how to write a formula for a_n

4. Series: $\sum_{n=1}^{\infty} a_n$

   (a) Know the definition of a series.

   (b) What's the difference between a series and sequence.

   (c) What's a partial sum? Given $S = \sum_{n=1}^{\infty} a_n$ and $S_N = \sum_{n=1}^{N} a_n$, what is $\lim_{n\to\infty} a_n$, $\lim_{n\to\infty} s_n$?

   (d) If you're asked to find the sum of a series, check to see if it's geometric. It's the only series we covered that we can find the sum.

   (e) Can you come up with a couple of convergent series and a couple of divergent series?

5. Convergence Tests for $\sum a_n$

   (a) Divergence Test: Check to see if $\lim_{n\to\infty} a_n = 0$? If it's not 0, the series diverges. If it **IS** 0, then you move on to a different test.

   (b) Integral Test. Look at the sequence. If it looks easily integrable, try this test. For example given $\sum \frac{n}{1+n^2}$, it looks like $\frac{n}{1+n^2}$ can be integrated using u-sub.

   (c) Geometric Series: Can the series be written as $\sum_{n=0}^{\infty} ar^n$? If $-1 < r < 1$, then the series converges and $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$. Otherwise it diverges.

(d) p-test: Is the series of the form $\sum \frac{1}{n^p}$. If $p > 1$, then the series converges. You'll use p-series when using DCT and LCT with another series.

(e) Direct Comparison Test. Know the statement of the test. You can find this in the stategy sheet I handed out. If your sequence a_n looks like a rational function (ex. $\frac{n^2}{n^4-1}$ or $\frac{\sqrt{n+1}}{2n}$), then use a comparison test with a p-series.

(f) Limit Comparison Test. Same as above.

(g) Alternating series Test. Know the statement and conditions of the test. If you converge by AST, then check $\sum |a_n|$ to see if it's absolutely or conditionally convergent.

(h) A series $\sum a_n$ is absolutely convergent if $\sum a_n$ converges and $\sum |a_n|$ is convergent.

(i) A series $\sum a_n$ is conditionally convergent if $\sum a_n$ converges but $\sum |a_n|$ is divergent.

(j) Ratio Test. Use this test if you see exponentials or factorials (a^n, $n!$, $(2n)!$, etc.)

(k) Root Test. Given $\sum a_n$, if a_n is of the form $a_n = (b_n)^n$, then try the Root Test.

6. Power Series

(a) Has the form: $\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + ...$

(b) Know how to find the center, radius of convergence, and interval of convergence.

(c) By default, use the Ratio Test to find the interval of convergence.

(d) Use the Root Test for the same reasons I recommended using the root test from the previous section.

(e) Remember $|x-a| < R \quad \rightarrow \quad -R < x-a < R$

(f) If $R \neq 0$ and $R \neq \infty$, then you need to manually check the endpoints. For example suppose you do the ratio test and find the interval of convergence is $-1 < x < 4$. You need to plug $x = -1$ and $x = 4$ into the power series and check its convergence.

7. YouTube is an excellent resource if you're looking for examples. I found this video from PatrickJMT. He goes over 14 series and explains his process on how he chooses his tests.

`http://patrickjmt.com/strategy-for-testing-series-practice-problems/`

NOTE: His answer to #8 is incorrect. The series will diverge. Confirm this by doing the Ratio Test.