Math 230 Calculus II

Practice problems for Exam III

Exam III will be based on Sections 7.8, 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.8

1. Review all definitions from Sections 7.8, 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.8
2. Review all theorems from Sections 7.8, 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.8
3. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   (a) $\int_0^\infty \frac{1}{\sqrt[4]{1+x}}\,dx$. *Answer:* Divergent.

   (b) $\int_{-\infty}^0 2^r\,dr$. *Answer:* $1/\ln 2$.

   (c) $\int_{-\infty}^\infty xe^{-x^2}\,dx$. *Answer:* 0.

   (d) $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$. *Answer:* $2e^{-1}$.

   (e) $\int_1^\infty \frac{1}{x^2+x}\,dx$. *Answer:* $\ln 2$.

   (f) $\int_1^\infty \frac{\ln x}{x}\,dx$. *Answer:* Divergent.

   (g) $\int_0^1 \frac{3}{x^5}\,dx$. *Answer:* Divergent.

   (h) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$. *Answer:* $\pi/2$.

4. Sketch the region and find its area (if the area is finite).

   (a) $S = \{(x,y) \mid x \geq 1,\ 0 \leq y \leq e^{-x}\}$. *Answer:* $1/e$.

   (b) $S = \{(x,y) \mid x \leq 0,\ 0 \leq y \leq e^{x}\}$. *Answer:* 1.

5. Determine whether the integral is convergent or divergent. Uses Direct Comparison Test.

   (a) $\int_0^\infty \frac{x}{x^3+1}\,dx$. *Answer:* Convergent.

   (b) $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$. *Answer:* Divergent.

   (c) $\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$. *Answer:* Convergent.

6. What is a sequence? What does it mean to say that $\lim_{n\to\infty} a_n = 8$?
7. Give two example of convergent sequences. Give two examples of divergent sequences.
8. Determine whether the sequence is convergent or divergent. If it is convergent, find the limit.

   (a) $a_n = \frac{n^3}{n^3+1}$. *Answer:* 1.

   (b) $a_n = e^{1/n}$. *Answer:* 1.

   (c) $a_n = \frac{3^{n+2}}{5^n}$. *Answer:* 0.

   (d) $a_n = \sqrt{\frac{n+1}{9n+1}}$. *Answer:* $1/3$.

   (e) $a_n = e^{2n/(n+2)}$. *Answer:* e^2.

   (f) $a_n = \frac{n^3}{n+1}$. *Answer:* Divergent.

   (g) $a_n = \cos(2/n)$. *Answer:* 1.

   (h) $a_n = n^2e^{-n}$. *Answer:* 0.

   (i) $a_n = \frac{\cos^2 n}{2^n}$. *Answer:* 0.

   (j) $a_n = 2^{-n}\cos n\pi$. *Answer:* 0.

   (k) $a_n = \frac{(\ln n)^2}{n}$. *Answer:* 0.

   (l) $a_n = \tan^{-1}(\ln n)$. *Answer:* $\pi/2$.

9. What is the difference between a sequence and a series?

10. Explain what it means to say that $\sum_{n=1}^{\infty} a_n = 5$.

11. Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$ whose partial sums are given.

(a) $s_n = 2 - 3(0.8)^n$. *Answer:* 2.

(b) $s_n = \dfrac{n^2 - 1}{4n^2 + 1}$. *Answer:* 1/4.

12. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}}$. *Answer:* Divergent.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$. *Answer:* 1/7.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{3n-1}$. *Answer:* Divergent.

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$. *Answer:* $\frac{3e}{3-e}$.

(e) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$. *Answer:* Divergent.

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{2}{3}\right)^n}$. *Answer:* Divergent.

(g) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$. *Answer:* 3/2.

(h) $\displaystyle\sum_{n=1}^{\infty} \tan^{-1} n$. *Answer:* Divergent.

13. Determine whether the series is convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$. *Answer:* Divergent.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4}$. *Answer:* Convergent.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+6n+13}$. *Answer:* Convergent.

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$. *Answer:* Divergent.

(e) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$. *Answer:* Convergent.

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$. *Answer:* Divergent.

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$. *Answer:* Divergent

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n\sin^2 n}{1+n^3}$. *Answer:* Convergent.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$. *Answer:* Divergent.

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{1+4^n}{1+3^n}$. *Answer:* Divergent.

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$. *Answer:* Convergent.

(l) $\displaystyle\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$. *Answer:* Convergent.

(m) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$. *Answer:* Convergent.

(n) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$. *Answer:* Convergent.

(o) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$. *Answer:* Convergent.

14. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+4}$.
Answer: Conditionally convergent.

(b) $\sum_{n=1}^{\infty}\frac{n!}{100^n}$. *Answer:* Divergent.

(c) $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{\sqrt{n^3+2}}$.
Answer: Conditionally convergent.

(d) $\sum_{n=1}^{\infty}\frac{n^{10}}{(-10)^{n+1}}$.
Answer: Absolutely convergent.

(e) $\sum_{n=1}^{\infty}\frac{(-2)^n}{n^n}$. *Answer:* Absolutely convergent.

(f) $\sum_{n=1}^{\infty}\left(\frac{n^2+1}{2n^2+1}\right)^n$.
Answer: Absolutely convergent.

(g) $\sum_{n=2}^{\infty}\left(\frac{-2n}{n+1}\right)^{5n}$. *Answer:* Divergent.

(h) $\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$. *Answer:* Divergent.

15. Determine the radius of convergence and the interval of convergence of the series.

(a) $\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n^2}$. *Answer:* $R=1$; $[-1,\ 1]$.

(b) $\sum_{n=0}^{\infty}\frac{x^n}{n!}$. *Answer:* $R=\infty$; $(-\infty,\ \infty)$.

(c) $\sum_{n=1}^{\infty}\frac{n^n}{x^n}$. *Answer:* $R=0$; $\{0\}$.

(d) $\sum_{n=1}^{\infty}\frac{10^n x^n}{n^3}$. *Answer:* $R=\frac{1}{10}$; $(-\frac{1}{10},\ \frac{1}{10})$.

(e) $\sum_{n=0}^{\infty}\frac{(-1)^n(x-3)^n}{2n+1}$. *Answer:* $R=1$; $(2,\ 4]$.

(f) $\sum_{n=1}^{\infty}\frac{3^n(x+4)^n}{\sqrt{n}}$. *Answer:* $R=\frac{1}{3}$; $[-\frac{13}{3},-\frac{11}{3})$.

16. Find a power series representation for the function and determine the interval of convergence.

(a) $\frac{1}{1+x}$. *Answer:* $\sum_{n=0}^{\infty}(-1)^n x^n$; $(-1,\ 1)$.

(b) $\frac{5}{1-4x^2}$. *Answer:* $5\sum_{n=0}^{\infty}4^n x^{2n}$; $(-\frac{1}{2},\ \frac{1}{2})$.

(c) $\frac{1}{x+10}$. *Answer:* $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{10^{n+1}}$; $(-10,\ 10)$.

(d) $\frac{x}{9+x^2}$. *Answer:* $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{9^{n+1}}$; $(-3,\ 3)$.