This study guide is in no way exhaustive. As stated in class, any type of question from class, quizzes, exams, and homeworks are fair game. Every problem can have a variation or require multiple techniques.

1. **Some Algebra Review**

   (a) **Logarithm Properties**

      i. $y = \log_b x$ is equivalent to $x = b^y$.

      ii. $\log_b b = 1$

      iii. $\log_b 1 = 0$, $\ln 1 = 0$

      iv. $\log_b(b^r) = r$, $\ln(e^r) = r$

      v. $\log_b(x^r) = r\log_b(x)$, $\ln(x^r) = r\ln(x)$

      vi. $\log_b(MN) = \log_b(M) + \log_b(N)$

      vii. $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

      viii. The domain of $\log_b(u)$ is $u > 0$

      ix. All the rules above hold for $\ln x$. Remember that $\ln x = \log_e x$

      x. Change of Base

      $$\log_a(M) = \frac{\ln M}{\ln a} = \frac{\log_b M}{\log_b a}$$

      This is useful if you're asked to find the derivative of $y = \log_5(x+1)$

   (b) **Exponential Properties**

      i. $a^n a^m = a^{n+m}$

      ii. $(a^n)^m = a^{nm}$

      iii. $(ab)^n = a^n b^n$

      iv. $\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

      v. $a^{-n} = \frac{1}{a^n}$

      vi. $\frac{1}{a^{-n}} = a^n$

      vii. $a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$

   (c) **Properties of Radicals**

      i. $\sqrt[n]{a} = a^{1/n}$

      ii. $\sqrt[n]{a^m} = a^{m/n}$

2. **Derivatives**

   (a) **Limit Definition of a Derivative**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \text{or} \quad f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

   (b) **Tangent Lines**

   When asked to find the equation of a tangent line on f at $x = a$, you need two things: A point and a slope.

   i. You're usually given the x value. If they don't tell you the y value, you must plug the x value into $f(x)$ to get the y value. Now you have a point $(a, f(a))$

   ii. To find the slope m, you find $m = f'(a)$.

   iii. The equation of the tangent line is $y - f(a) = f'(a)(x - a)$. You may need to write this in slope-intercept form.

   (c) **Derivative Formulas**

   i. $\frac{d}{dx}(c) = 0$

   ii. $\frac{d}{dx}(f \pm g) = f' \pm g'$

   iii. $\frac{d}{dx}(x) = 1$

   iv. $\frac{d}{dx}(kx) = k$

   v. Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

   vi. $\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1} \cdot f'(x)$

   vii. Product Rule: $(f\ g)' = f'g + fg'$

   viii. Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

   ix. Chain Rule: $(f(g(x)))' = f'(g(x)) \cdot f'(x)$

   x. $\frac{d}{dx}(a^x) = a^x \ln a$

   xi. $\frac{d}{dx}(e^x) = e^x$

   xii. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

   xiii. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

   xiv. $\frac{d}{dx}\left(e^{f(x)}\right) = f'(x) \cdot e^{f(x)}$

   xv. $\frac{d}{dx}(\ln(f(x))) = \frac{1}{f(x)} \cdot f'(x)$

   xvi. $\frac{d}{dx}(\sin x) = \cos x$

   xvii. $\frac{d}{dx}(\cos x) = -\sin x$

   xviii. $\frac{d}{dx}(\tan x) = \sec^2 x$

   xix. $\frac{d}{dx}(\sec x) = \sec x\ \tan x$

   xx. $\frac{d}{dx}(\csc x) = -\csc x\ \cot x$

   xxi. $\frac{d}{dx}(\cot x) = -\csc^2 x$

   REMEMBER: all of these derivatives have a version for the chain rule. For example:

$$\frac{d}{dx}\left(\tan^{-1}(e^x)\right) = \frac{1}{1+(e^{2x})^2} \cdot 2e^{2x}$$

   (d) **Inverse Derivative Theorem**

   Let $f(x)$ be a one-to-one function.

$$\left(f^{-1}(b)\right)' = \frac{1}{f'(f^{-1}(b))}$$

3. **Integrals**

   (a) **Definitions**

      i. **Antideriative:** An antiderivative of $f(x)$ is a function $F(x)$, where $F'(x) = f(x)$.

      ii. **General Antiderivative:** The general antiderivative of $f(x)$ is $F(x) + C$, where $F'(x) = f(x)$. Also known as the **Indefinite Integral**

$$\int f(x)\,dx = F(x) + C$$

      iii. **Definite Integral:**

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

   (b) **Common Integrals**

      i. $\displaystyle\int k\,dx = kx + C$

      ii. $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

      iii. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

      iv. $\displaystyle\int \frac{1}{kx+b}\,dx = \frac{1}{k}\ln|kx+b| + C$

      v. $\displaystyle\int e^x\,dx = e^x + C$

      vi. $\displaystyle\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

      vii. $\displaystyle\int e^{kx+b}\,dx = \frac{1}{k}e^{kx+b} + C$

      viii. $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C$

      ix. $\displaystyle\int a^{kx}\,dx = \frac{1}{k\ln a}a^{kx} + C$

      x. $\displaystyle\int a^{kx+b}\,dx = \frac{1}{k\ln a}a^{kx+b} + C$

      xi. $\displaystyle\int \ln x\,dx = x\ln x - x + C$

      xii. $\displaystyle\int \cos x\,dx = \sin x + C$

      xiii. $\displaystyle\int \sin x\,dx = -\cos x + C$

      xiv. $\displaystyle\int \sec^2 x\,dx = \tan x + C$

      xv. $\displaystyle\int \csc^2 x\,dx = -\cot x + C$

      xvi. $\displaystyle\int \sec x\tan x\,dx = \sec x + C$

      xvii. $\displaystyle\int \csc x\cot x\,dx = -\csc x + C$

      xviii. $\displaystyle\int \tan x\,dx = \ln|\sec x| + C$

      xix. $\displaystyle\int \cot x\,dx = \ln|\sin x| + C$

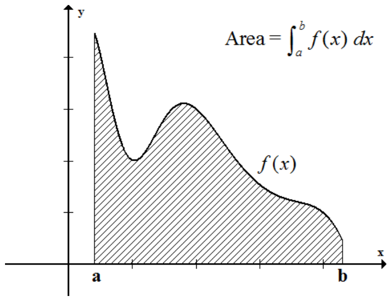
   (c) Every integral must be written into the proper form in order to use the formulas. For example,

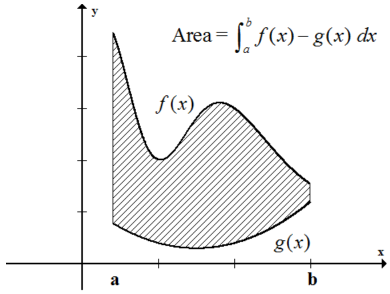
$$\int \sqrt{x}\,dx = \int x^{1/2}\,dx$$

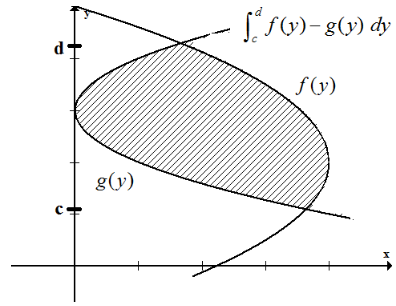
$$\int \frac{3}{5x^4}\,dx = \int \frac{3}{5}x^{-4}\,dx$$

$$\int \frac{8}{\sqrt[5]{x^9}}\,dx = \int 8x^{-9/5}\,dx$$

(d) Finding Area under or between Curves

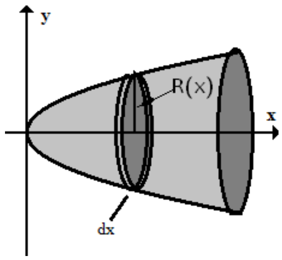






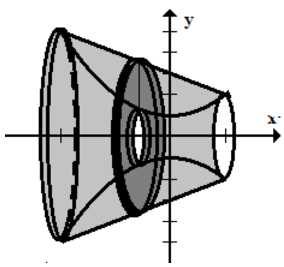
4. **Solids of Revolution**

(a) Disk Method - in terms of x



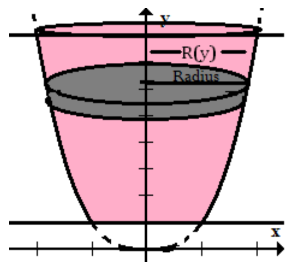
$$V = \int_a^b \pi R(x)^2 \, dx$$

(b) Washer Method - in terms of x



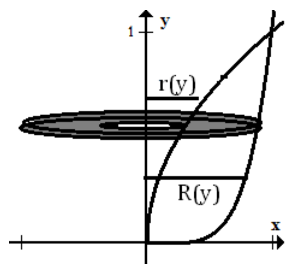
$$V = \int_a^b \pi R(x)^2 - \pi r(x)^2 \, dx$$

(c) Disk Method - in terms of y



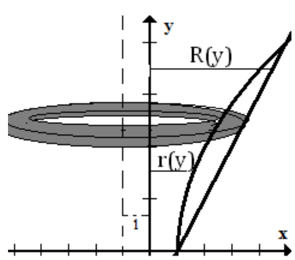
$$V = \int_c^d \pi R(y)^2 \, dy$$

(d) Washer Method - in terms of y



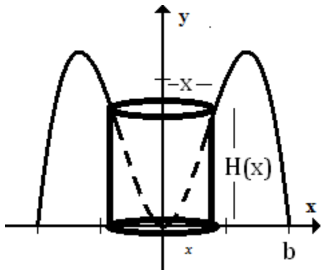
$$V = \int_c^d \pi R(y)^2 - \pi r(y)^2 \, dy$$

You may have to rotate around an axis other than the x or y axis. If you do, you need to adjust the radius. For example,



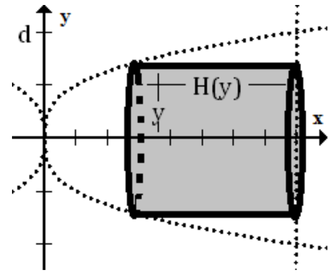
$$V = \int_c^d \pi(1 + R(y))^2 + \pi(1 + r(y))^2 \, dy$$

(a) Shell - in terms of x



$$V = \int_a^b 2\pi x H(x)\, dx$$

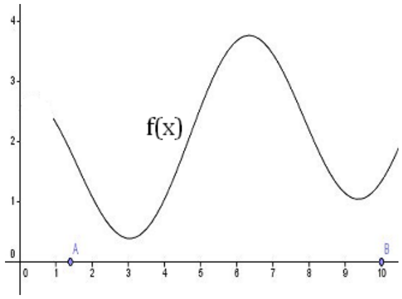
(b) Shell - in terms of y



$$V = \int_c^d 2\pi y H(y)\, dy$$

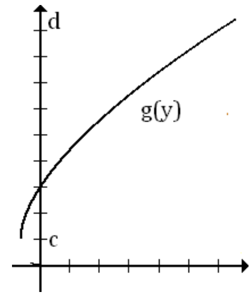
5. Arc Length

(a) In terms of x



$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

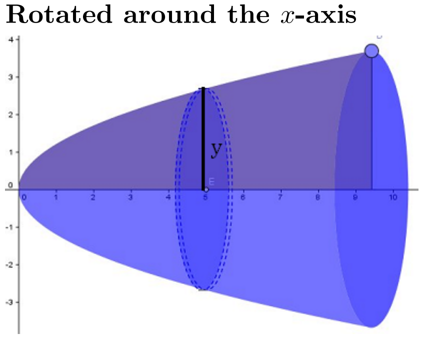
(b) In terms of y



$$L = \int_c^d \sqrt{1 + (g'(x))^2}\, dy$$

6. **Area of a Surface of Revolution**

(a) **Rotated around the x-axis**



$$SA = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad SA = \int_c^d 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

The first integral is in terms of x. That means the y in front of the square root should be in terms of x. For example, $y = x^2 + 2$. You use $x^2 + 2$ instead of y.

(b) **Rotated around the y-axis.**

The formula doesn't change much.

$$SA = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad SA = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

The second integral is in terms of y. That means the x in front of the square root should be in terms of y. For example, if $y = x^2$, solve for x ($x = \sqrt{y}$) and use $\sqrt{y}$ instead of x.

7. **Integration Techniques**

   (a) u **Substitution**

   Given $\int_a^b f(g(x))g'(x)\,dx$,

   i. Let $u = g(x)$

   ii. Then $du = g'(x)\,dx$

   iii. If there are bounds, you must change them using $u = g(b)$ and $u = g(a)$

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$