Math 230

Calculus II

Brian Veitch • Fall 2015 • Northern Illinois University

Strategy for Integration

Formula 1: Integrals So far!

1. $\int k\, dx = kx + C$
2. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{x}\, dx = \ln|x| + C$
4. $\int \frac{1}{kx+b}\, dx = \frac{1}{k}\ln|kx+b| + C$
5. $\int e^x\, dx = e^x + C$
6. $\int e^{kx}\, dx = \frac{1}{k}e^{kx} + C$
7. $\int a^x\, dx = \frac{a^x}{\ln a} + C$
8. $\int a^{kx}\, dx = \frac{1}{k\ln a}a^{kx} + C$
9. $\int \ln x\, dx = x\ln x - x + C$
10. $\int \cos x\, dx = \sin x + C$
11. $\int \sin x\, dx = -\cos x + C$
12. $\int \sec^2 x\, dx = \tan x + C$
13. $\int \csc^2 x\, dx = -\cot x + C$
14. $\int \sec x \tan x\, dx = \sec x + C$
15. $\int \csc x \cot x\, dx = -\csc x + C$
16. $\int \tan x\, dx = \ln|\sec x| + C$
17. $\int \cot x\, dx = \ln|\sin x| + C$
18. $\int \csc x\, dx = \ln|\csc x - \cot x| + C$
19. $\int \sec x\, dx = \ln|\sec x + \tan x| + C$
20. $\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
21. $\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
22. $\int -\frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
23. $\int \frac{1}{x\sqrt{x^2 - 1}}\, dx = \sec^{-1}(x) + C$
24. $\int -\frac{1}{x\sqrt{x^2 - 1}}\, dx = \csc^{-1}(x) + C$

Strategies

1. Simplify the integrand, if possible

   (a) Ex. $\displaystyle\int \frac{\tan(x)}{\sec^2(x)}\,dx = \int \sin(x)\cos(x)\,dx = \frac{1}{2}\int \sin(2x)\,dx$

   (b) Ex. $\displaystyle\int \sqrt{x}(1+\sqrt{x})\,dx = \int \sqrt{x} + x\,dx$

   (c) Ex. $\displaystyle\int \frac{1+x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$

2. Look for obvious substitutions.

   (a) Try letting u equal expressions that are in parentheses, where $\dfrac{du}{dx}$ is also in the integrand.

$$\text{Ex.} \quad \int x(x^2+1)\,dx, \ \text{Let } u = x^2+1$$

$$\text{Ex.} \quad \int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx, \ \text{Let } u = \sqrt{x}$$

3. Try to classify the integrand according to its form.

   (a) Basic u-substitution (but that's already been covered).

   (b) Integration by Parts

   If the integrand is a product of a power of x (or some polynomial) and a function (trig functions, exponential functions, log functions).

   (c) Rational Functions

   If the denominator of the rational function can be factored easily, give partial fractions a shot.

(d) Powers of trig functions

If your integrand has the form $\sin^m(x)\cos^n(x)$ or $\tan^m(x)\sec^n$, use techniques from that section.

(e) If you have integrands with denominators of

$$\sqrt{a^x - x^2}, a^2 + x^2, \sqrt{x^2 - a^2}$$

try trig substitution.

4. Keep trying. There are basically two methods for integration: substitution and parts. If the method isn't obvious, try any of the above methods just to see what happens.

(a) Try substitution again

(b) Try integration by parts

(c) Manipulate the integrand (simplify, conjugates, rewriting trig functions, identities, etc.)

(d) Try multiple methods. You've probably come across problems that require multiple methods.

$$\int e^{\sqrt{x}}\, dx$$

Let $u = \sqrt{x}$, $du = \dfrac{1}{2\sqrt{x}}\, dx$. Notice that this doesn't help. The substitution method failed. Try another substitution.

Let $u^2 = x \to 2\, du = dx$

$$\int e^{\sqrt{x}}\, dx = \int 2ue^u\, du$$

Now try integration by parts.