Math 230, Calculus II, Spring 2016 Name: SOLUTION

Recall the anti-derivative of $f(x) = ax^n$ is $F(x) = \dfrac{ax^{n+1}}{n+1}$.
Write your final answer on the line given. Attach **ALL** work (on loose-leaf paper) to the back of this worksheet.

1. Find the general anti-derivative $F(x)$ to the following functions

   (a) $f(x) = 4x + 7 - \dfrac{10}{x^9}$ $\quad F(x) = 2x^2 + 7x + \frac{5}{4}x^{-8} + C$

   (b) $f(\theta) = 3\cos(\theta) - 4\sin(\theta)$ $\quad F(\theta) = 3\sin\theta + 4\cos\theta + C$

   (c) $f(x) = 8\sqrt{x} - \sec(x)\tan(x)$ $\quad F(x) = 16/3\, x^{3/2} - \sec x + C$

   (d) $f(x) = 7x^{2/5} + 9x^{-1/5}$ $\quad F(x) = 5x^{7/5} + 45/4\, x^{4/5} + C$

2. Another way to ask for the anti-derivative of $f(x)$ is to use the indefinite integral $\int f(x)\, dx$

   (a) $\int x^{1.3} + 7x^{2.5}\, dx$ $\quad = \frac{10}{23}x^{2.3} + 2x^{3.5} + C$

   (b) $\int \sqrt[4]{x^5} - 3x^3\, dx$ $\quad = 4/9\, x^{9/4} - 3/4\, x^4 + C$

   (c) $\int (x+4)(2x-1)\, dx$ $\quad = 2/3\, x^3 + 7/2\, x^2 - 4x + C$

   (d) $\displaystyle\int \frac{1 + \sqrt{x} + x}{\sqrt{x}}\, dx$ $\quad = 2x^{1/2} + x + 2/3\, x^{3/2} + C$

   (e) $\displaystyle\int \frac{1 - \cos^3(\theta)}{\cos^2(\theta)}\, d\theta$ $\quad = \tan\theta - \sin\theta + C$

3. We also have definite integrals: $\displaystyle\int_a^b f(x)\, dx = F(x)\big|_a^b = F(b) - F(a)$

   (a) $\displaystyle\int_{-2}^{3} x^3 - 3x + 7\, dx$ $\quad = 175/4$

   (b) $\displaystyle\int_0^{\pi} 4\sin(\theta) - 3\cos(\theta)\, d\theta$ $\quad = 8$

   (c) $\displaystyle\int_1^2 \frac{1}{x^2} + \frac{4}{x^3}\, dx$ $\quad = 2$

   (d) $\displaystyle\int_{\pi/6}^{\pi/2} \csc(x)\cot(x)\, dx$ $\quad = 1$

4. The last section we covered was integration by substitution. Given $\int f(g(x))g'(x)\, dx$, use the following substitution $u = g(x)$ and $du = g'(x)\, dx$.

$$\int f(g(x))g'(x)\, dx = \int f(u)\, du$$

(a) $\int 3x^2(x^3+4)^5\,dx$

$u =$ x^3+4

$du =$ $3x^2dx$

Answer: $\frac{1}{6}(x^3+4)^6 + C$

(b) $\int \sqrt{4x-5}dx$

$u =$ $4x-5$

$du =$ $4dx$

Answer: $\frac{1}{6}(4x-5)^{3/2} + C$

(c) $\int \frac{x}{\sqrt{x^2+1}}\,dx$

$u =$ x^2+1

$du =$ $2x\,dx$

Answer: $(x^2+1)^{1/2} + C$

5. When evaluating a definite integral using substitution you must also change your bounds a and b to $g(a)$ and $g(b)$

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

(a) $\int_1^2 \frac{2}{x^2}\left(1+\frac{1}{x}\right)^3\,dx$ $=$ $175/32$

(b) $\int_0^{\pi/6} 4\tan(\theta)\sec^2(\theta)\,d\theta$ $=$ $2/3$