MATH 230 NAME:
8.1

Directions: Show all work on a separate sheet of paper for full credit.

1. Use the arc length formula to find the length of the curve $y = 2x - 5$, $-1 \le x \le 3$.

$$\int_{-1}^{3} \sqrt{1 + 2^2}\, dx = 4\sqrt{5}$$

2. Use the arc length formula to find the length of the curve $y = \sqrt{2 - x^2}$, $0 \le x \le 1$. Check your answer by noting that the curve is part of a circle.

$$\int_0^1 \sqrt{1 + \frac{x^2}{1 - x^2}}\, dx = \int_0^1 \sqrt{\frac{1}{1 - x^2}}\, dx = \int_0^1 \frac{1}{\sqrt{1 - x^2}} = \sin^{-1} x \Big|_0^1 = \pi/4$$

3. Find the exact length of the curve.

(a) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $1 \le x \le 2$

$$\int_1^2 \sqrt{\left(\frac{1}{2}x + \frac{1}{2x}\right)^2}\, dx = \frac{3}{4} + \frac{1}{2}\ln 2$$

(b) $y = \ln(\cos x)$, $0 \le x \le \pi/3$

$$\int_0^{\pi/3} \sec x\, dx = \ln(2 + \sqrt{3})$$

(c) $y = 1 + 6x^{3/2}$, $0 \le x \le 1$

$$\int_0^1 \sqrt{1 + 81x}\, dx = \frac{2}{243}(82^{3/2} - 1)$$