

MATH 230
HW 18 (7.8)

Directions: Show all work on a separate sheet of paper for full credit.

1. Which of the following integrals are improper? Why?

- (a) $\int_0^{\pi/4} \tan(x) dx$ Proper
- (b) $\int_0^{\pi} \tan(x) dx$ Improper Type II
- (c) $\int_{-1}^1 \frac{dx}{x^2 - x - 2}$ Improper Type II
- (d) $\int_0^{\infty} e^{-x^2} dx$ Improper Type I

2. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

- (a) $\int_3^{\infty} \frac{1}{(x-2)^{3/2}} dx$ Converges to 2
- (b) $\int_0^{\infty} \frac{1}{\sqrt[4]{1+x}} dx$ Divergent
- (c) $\int_0^{\infty} \frac{x^2}{\sqrt{1+x^3}} dx$ Divergent
- (d) $\int_{-\infty}^{\infty} x e^{-x^2} dx$ Converges to 0
- (e) $\int_2^{\infty} y e^{-3y} dy$ Converges to $\frac{7}{9}e^{-6}$
- (f) $\int_1^{\infty} \frac{\ln(x)}{x} dx$ Divergent
- (g) $\int_1^{\infty} \frac{\ln(x)}{x^2} dx$ Converges to 1
- (h) $\int_{-2}^3 \frac{1}{x^4} dx$ Divergent
- (i) $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$ Converges to $\frac{32}{3}$

3. Sketch the region and find its area (if the area is finite).

$$S = \{(x, y) | x \geq 1, 0 \leq y \leq e^{-x}\}$$

$$\text{Answers: Area} = \int_1^{\infty} e^{-x} dx = e^{-1}$$