MATH 230 NAME:
(6.2, 6.3)

Directions: Show all work on a separate sheet of paper for full credit.

1. Use the Laws of Logarithms to expand the quantity $\ln \sqrt[3]{\dfrac{x-1}{x+1}}$.

$$= \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+1)$$

2. Express the quantity as a single logarithm: $2\ln(x) + 3\ln(y) - \ln(z)$.

$$\ln \frac{x^2y^3}{z}$$

3. Simplify the expression $e^{\ln(\ln(e^3))}$. 3

4. Find the limit.

   a) $\lim_{x\to\infty} e^{-x^2}$ $= 0$

   b) $\lim_{x\to\infty} e^{-2x}\cos(x)$ $= 0$ (Squeeze Theorem)

   c) $\lim_{x\to\infty} [\ln(2+x) - \ln(1+x)]$ $= 0$ (Hint: Combine into one log)

5. Differentiate the function.

   a) $f(x) = x\ln(x) - x$ $\ln x$

   b) $f(x) = \sin(\ln(x))$ $\dfrac{\cos(\ln x)}{x}$

   c) $f(x) = \ln(\sin^2(x))$ $2\cot(x)$

   d) $f(y) = \ln \dfrac{(2y+1)^5}{\sqrt{y^2+1}}$ (Hint: try to expand using laws of logarithms first)

$$\frac{10}{2y+1} - \frac{y}{y^2+1}$$

   e) $f(x) = \ln(\ln(\ln(x)))$ $\dfrac{1}{\ln(\ln x)} \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x}$

   f) $f(x) = \ln(\csc(x) - \cot(x))$ $\csc(x)$

   g) $f(x) = e^x + x^e$ $e^x + ex^{e-1}$

   h) $y = \dfrac{e^x}{1-e^x}$ $\dfrac{e^x}{(1-e^x)^2}$

   i) $f(t) = \sin(e^{3t^2})$ $6te^{3t^2}\cos(e^{3t^2})$

j) $f(x) = e^{x\sin(2x)}$ $e^{x\sin(2x)} \cdot (\sin(2x) + 2x\cos(2x))$

k) $f(x) = x^2 e^{-1/x}$ $e^{-1/x}(1 + 2x)$

6. Use logarithmic differentiation to differentiate

a) $y = (x^2+2)^2(x^4+4)^4$ $(x^2+2)^2(x^4+4)^4\left(\dfrac{4x}{x^2+2} + \dfrac{16x^3}{x^4+4}\right)$

b) $y = \sqrt{\dfrac{x-1}{x^4+1}}$ $\sqrt{\dfrac{x-1}{x^4+1}}\left(\dfrac{1}{2x-2} - \dfrac{2x^3}{x^4+1}\right)$

7. Evaluate the integral.

a) $\displaystyle\int_0^3 \frac{dx}{5x+1}dx$ $\dfrac{1}{5}\ln 16$

b) $\displaystyle\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$ $\int_4^9 x + 2 + \dfrac{1}{x}\,dx = 85/2 + \ln(9/4)$

c) $\displaystyle\int \frac{dx}{x\ln(x)}$ $\ln(\ln x) + C$

d) $\displaystyle\int \frac{\cos(\ln(t))}{t}dt$ $\sin(\ln t) + C$

e) $\displaystyle\int_0^1 (x^e + e^x)dx$ $\dfrac{1}{e+1} + e - 1$

f) $\displaystyle\int e^{\tan(x)}\sec^2(x)dx$ $e^{\tan x} + C$

g) $\displaystyle\int \frac{(1+e^x)^2}{e^x}dx$ $-e^{-x} + 2x + e^x + C$ (Distribute numerator and simplify)

h) $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$ $\dfrac{1}{3}(\ln x)^3 + C$

i) $\displaystyle\int \frac{\cos x}{2+\sin x}\,dx$ $\ln|2 + \sin x| + C$