MATH 230 NAME:
5.3

Directions: Show all work on a separate sheet of paper for full credit.

1. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves around the given line.

   (a) $y = x^3$, $y = 0$, $x = 1$, and $x = 2$ about the y-axis. $\int_1^2 2\pi x \cdot x^3 \, dx = 62\pi/5$

   (b) $y = x^2$, $0 \le x \le 2$, $y = 4$, $x = 0$ about the y-axis. $\int_0^2 2\pi x(4 - x^2) \, dx = 8\pi$

   (c) $y = \sqrt{x}$, $x = 0$, and $y = 2$ about the x-axis. $\int_0^2 2\pi y \cdot y^2 \, dy = 8\pi$

   (d) $y = 1/x$, $x = 0$, $y = 1$, $y = 3$ about the x-axis. $\int_1^3 2\pi y \cdot 1/y \, dy = 4\pi$

   (e) $y = 4x - x^2$, $y = 3$ about $x = 1$. $\int_1^3 2\pi(x - 1)(-x^2 + 4x - 3) \, dx = 8\pi/3$

   (f) $y = 4 - 2x$, $y = 0$, $x = 0$; about $x = -1$ $\int_0^2 2\pi(x + 1)(4 - 2x) \, dx = 40\pi/3$

   (g) $x = 2y^2$, $x = y^2 + 1$ about $y = -2$. $\int_{-1}^1 2\pi(y + 2)(1 - y^2) \, dy = 16\pi/3$

2. Let V be the volume of the solid obtained by rotating about the y axis the region bounded by $y = \sqrt{x}$ and $y = x^2$. Find V both by Washers and Shells.

$$V = \int_0^1 \pi \left[(\sqrt{y})^2 - (y^2)^2 \right] \, dy = \frac{3\pi}{10}$$

$$V = \int_0^1 2\pi x \left(\sqrt{x} - x^2 \right) \, dx = \frac{3\pi}{10}$$