MATH 230
11.9

Directions: Show all work on a separate sheet of paper for full credit.

1. If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 10, what is the radius of convergence of the series $\sum_{n=1}^{\infty} n c_n x^{n-1}$? Why?

   answer: Also has a radius of convergence of 10. Look at Theorem 2 from the textbook. Taking a derivative of a series will not change the radius of convergence.

2. Suppose you know that the series $\sum b_n x^n$ converges for $|x| < 2$. What can you say about the following series? Why?

$$\sum \frac{b_n}{n+1} x^{n+1}$$

   answer: Note that $\sum \frac{b_n}{n+1} x^{n+1} = \int \sum b_n x^n \, dx$. We have a theorem that states these must have the same radius of convergence. The interval might be different since the new series might converge at the endpoints.

3. Find a power series representation for the function and determine the interval of convergence.

   (a) $f(x) = \dfrac{1}{1+x}$ $\qquad \sum_{n=0}^{\infty} (-1)^n x^n$ with $I = (-1, 1)$

   (b) $f(x) = \dfrac{5}{1-4x^2}$ $\qquad \sum_{n=0}^{\infty} 5 \cdot 4^n x^{2n}$ with $I = (-1/2, 1/2)$

   (c) $f(x) = \dfrac{x^2}{x^4+16}$ $\qquad \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{4n+2}}{2^{4n+4}}$ with $I = (-2, 2)$

   (d) $f(x) = \dfrac{4}{2x+3}$ $\qquad \sum_{n=0}^{\infty} (-1)^n \dfrac{2^{n+2}}{3^{n+1}} x^n$ with $I = (-3/2, 3/2)$

4. Find a power series representation for $f(x) = \ln(5 - x)$ and determine the radius of convergence.

   answer: $\sum_{n=0}^{\infty} -\dfrac{x^{n+1}}{(n+1)5^{n+1}} + C$ with $I = (-5, 5)$ and $R = 5$

5. Evaluate the indefinite integral $\int \dfrac{\tan^{-1}(x)}{x} dx$ as a power series. What is the radius of convergence?

   answer: $\dfrac{\tan^{-1} x}{x} = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{2n+1}$ so

$$\int \frac{\tan^{-1} x}{x} \, dx = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)^2} + C \text{ with } I = (-1, 1) \text{ and } R = 1$$

6. Find the power series representation of $\int \dfrac{x^2}{1+x^4} \, dx$. Use this to estimate $\int_0^{0.3} \dfrac{x^2}{1+x^4} \, dx$

   answer: $\dfrac{x^2}{1+x^4} = \sum_{n=0}^{\infty} (-1)^n x^{4n+2}$

so $\displaystyle\int_0^{0.3} \frac{x^2}{1+x^4}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{4n+3}\Bigg|_0^{0.3} = \sum_{n=0}^{\infty} \frac{(-1)^n (.3)^{4n+3}}{4n+3}$

You can add up the first few terms (say S_2) and we are accurate to within 6 decimal places.