

Directions: Show all work on a separate sheet of paper for full credit.

1. If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 10, what is the radius of convergence of the series $\sum_{n=1}^{\infty} n c_n x^{n-1}$? Why?

2. Suppose you know that the series $\sum b_n x^n$ converges for $|x| < 2$. What can you say about the following series? Why?

$$\sum \frac{b_n}{n+1} x^{n+1}$$

3. Find a power series representation for the function and determine the interval of convergence.

(a) $f(x) = \frac{1}{1+x}$

(b) $f(x) = \frac{5}{1-4x^2}$

(c) $f(x) = \frac{x^2}{x^4+16}$

(d) $f(x) = \frac{4}{2x+3}$

4. Find a power series representation for $f(x) = \ln(5-x)$ and determine the radius of convergence.

5. Evaluate the indefinite integral $\int \frac{\tan^{-1}(x)}{x} dx$ as a power series. What is the radius of convergence?

6. Find the power series representation of $\int \frac{x^2}{1+x^4} dx$. Use this to estimate $\int_0^{0.3} \frac{x^2}{1+x^4} dx$