MATH 230
11.7

Directions: Show all work on a separate sheet of paper for full credit.

1. Test the series for convergence or divergence. You may use any test from 11.2-11.6. If you find convergence using the alternating series test, determine if it's absolutely or conditionally convergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$ — Converges by LCT with $\displaystyle\sum \frac{1}{n^2}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ — Diverges by Integral Test with $\int \frac{1}{x \ln x}\, dx$

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2-1}{n^3+1}$ — Cond. convergent. Converges by AST but $\displaystyle\sum \frac{n^2-1}{n^3+1}$ diverges by LCT with $\displaystyle\sum \frac{1}{n}$

   (d) $\displaystyle\sum_{n=1} (-1)^{n-1} \frac{n^4}{4^n}$ — Absolutely convergent by Ratio Test.

   (e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{1}{3^n}\right)$ — Converges by p-test and geometric.

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ — Converges by Ratio Test

   (g) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^2-1}$ — Diverges because $\lim_{n\to\infty} a_n \neq 0$

   (h) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n)$ — Diverges by LCT with $\sum \frac{1}{n}$

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$ — Converges by Ratio Test

   (j) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ — Converges by Integral Test or DCT with $\int \frac{e^{1/x}}{x^2}\, dx$

   (k) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ — Converges by Root Test because $\lim \sqrt[n]{a_n} = 1/e < 1$.

   Hint: Rewrite $\left(\frac{n}{n+1}\right)^n$ as $\left(\frac{n+1}{n}\right)^{-n} = \left(1+\frac{1}{n}\right)^{-n}$. Show $\lim \left(1+\frac{1}{n}\right)^{-n} = e^{-1}$

   (l) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1} 3^{n+1}}{n^n}$ — Converges by Root Test