MATH 230
11.6

Directions: Show all work on a separate sheet of paper for full credit.

1. What can you say about the series $\sum a_n$ in each of the cases?

   (a) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 8$ — Diverges

   (b) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0.8$ — Converges

   (c) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$ — Inconclusive

2. Determine whether the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{5n+1}$ is absolutely convergent or conditionally convergent.

   answer: Converges AST but $\sum \frac{1}{5n+1}$ diverges by LCT with $\sum \frac{1}{n}$. So $\sum a_n$ coverges conditionally.

3. Determine whether the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ is absolutely convergent or conditionally convergent.

   answer: Converges by AST. Since $\sum \frac{1}{n^2+1}$ converges by (Integral Test or DCT with $\sum \frac{1}{n^2}$), the series is absolutely convergent.

4. Use the Ratio Test to determine whether the series is convergent or divergent.

   (a) $\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n+1)!}$ — Absolutely Convergent by Ratio Test. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$

   (b) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$ — Divergent by Ratio Test. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$

   (c) $\sum_{n=1}^{\infty} ne^{-n}$ — Absolutely Convergent by Ratio Test. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{e} < 1$

   (d) $\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$ — Absolutely Convergent by Ratio Test. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1/10 < 1$

   (e) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ — Divergent by Ratio Test. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 4 > 1$

5. Use the Root Test to determine whether the series is convergent or divergent.

   (a) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$ — Absolutely Convergent By Root Test. $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1/2 < 1$

   (b) $\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$. — Divergent by Root Test. $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 32 > 1$