

Directions: Show all work on a separate sheet of paper for full credit.

1. Test the series for convergence or divergence.

(a)  $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \frac{2}{11} - \dots$   $\sum \frac{(-1)^{n+1} \cdot 2}{2n+1}$  (Converges by AST)

(b)  $\frac{1}{\ln(3)} - \frac{1}{\ln(4)} + \frac{1}{\ln(5)} - \frac{1}{\ln(6)} + \frac{1}{\ln(7)} - \dots$   $\sum \frac{(-1)^{n+1}}{\ln(n+2)}$  (Converges by AST)

(c)  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$  Converges by AST

(d)  $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^2 + n + 1}$  Diverges by DT ( $\lim a_n \neq 0$ )

(e)  $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$  Converges by AST

(f)  $\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$  Converges by AST. Rewrite as  $\sum (-1)^{n+1} \frac{n}{e^n}$

Any series that converges by AST must show the three conditions:

- (a)  $b_n$  is decreasing. \*Show the derivative is negative.
- (b)  $b_n > 0$
- (c)  $\lim_{n \rightarrow \infty} b_n = 0$

2. How many terms of the series do we need to add together in order to find the sum to the indicated accuracy?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2 2^n} \text{ with } |\text{error}| < 0.0005$$

Find all the partial sums up to and including the partial sum that gives the indicated accuracy.

*answer:* Need  $N = 5$  terms.  $S_1 = \frac{1}{2}$ ,  $S_2 = 0.4375$ ,  $S_3 = 0.45139$ ,  $S_4 = 0.447482639$ , and  $S_5 = 0.448732639$ .  
Real sum is  $S \approx 0.448414206$