

MATH 230
HW 29 (11.11)

Directions: Show all work on a separate sheet of paper for full credit.

1. Let $f(x) = \sin x$

- (a) Find the 4th degree Taylor Polynomial centered at $a = 0$. Graph it and $f(x)$.
- (b) Evaluate f and the polynomial at $x = 0$, $x = \pi/4$, and $x = \pi/2$.

answer: $T_5(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5$. I trust you can evaluate $T_5(0)$, etc.

2. Find the Taylor polynomial $T_3(x)$ for the function f centered at the number a .

- (a) $f(x) = \cos(x)$ at $a = \pi/2$ $T_3(x) = -(x - \pi/2) + \frac{1}{6}(x - \pi/2)^3$
- (b) $f(x) = \tan^{-1}(x)$ at $a = 1$ $T_3(x) = \frac{\pi}{4} + \frac{1}{2}(x - 1) - \frac{1}{4}(x - 1)^2 + \frac{1}{12}(x - 1)^3$
- (c) $f(x) = e^{-x} \sin x$, $a = 0$ $T_3(x) = x - x^2 + \frac{1}{3}x^3$

3. (a) Approximate f by a Taylor Polynomial with degree n at a .

- (b) Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the given interval.

i. $f(x) = 1/x$, $a = 1$, $n = 2$, $0.7 \leq x \leq 1.3$.

answer: $T_2(x) = 1 - (x - 1) + (x - 1)^2$

$$|R_2(x)| \leq \frac{6/0.7^4}{6}(0.027) = 0.11245$$

ii. $f(x) = x \ln x$, $a = 1$, $n = 3$, $0.5 \leq x \leq 1.5$

answer: $T_3(x) = (x - 1) + \frac{1}{2}(x - 1)^2 - \frac{1}{6}(x - 1)^3$

$$|R_3(x)| \leq \frac{16}{24}(1/16) = 0.041667$$