Math 230
11.10

Directions: Show all work on a separate sheet of paper for full credit.

1. Use the definition of a Taylor Series to find the first four nonzero terms of the series for $f(x)$ centered at a.

   (a) $f(x) = \frac{1}{1+x}$, $a = 2$ $$\sum_{n=0}^{3} \frac{f^{(n)}(2)}{n!}(x-2)^n = \frac{1}{3} - \frac{1}{9}(x-2) + \frac{1}{27}(x-2)^2 - \frac{1}{81}(x-2)^3$$

   (b) $f(x) = \sqrt[3]{x}$, $a = 8$ $$\sum_{n=0}^{3} \frac{f^{(n)}(8)}{n!}(x-8)^n = 2 + \frac{1}{12}(x-8) - \frac{1}{188}(x-8)^2 + \frac{5}{20736}(x-8)^3$$

   (c) $f(x) = \sin(x)$, $a = \pi/6$ $$\sum_{n=0}^{3} \frac{f^{(n)}(\pi/6)}{n!}(x-\pi/6)^n = \frac{1}{2} + \frac{\sqrt{3}}{2}(x-\pi/6) - \frac{1}{4}(x-\pi/6)^2 - \frac{\sqrt{3}}{12}(x-\pi/6)^3$$

   (d) $f(x) = \ln x$, $a = 1$ $$\sum_{n=0}^{4} \frac{f^{(n)}(1)}{n!}(x-1)^n = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$$

2. Find the Maclaurin series for $f(x)$.

   (a) $f(x) = (1-x)^{-2}$ $$(1-x)^{-2} = \sum_{n=0}^{\infty}(n+1)x^n = \sum_{n=1}^{\infty} nx^{n-1}$$

   (b) $f(x) = 2^x$ $$2^x = \sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!}x^n$$

3. Find the Taylor series of $f(x) = \frac{1}{x}$ centered at $a = -3$.

   $$\frac{1}{x} = \sum_{n=0}^{\infty} \frac{-(x+3)^n}{3^{n+1}}$$

4. Use the Maclaurin series for $\sin x = \sum_{n=0} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$ to find the Maclaurin series for $f(x) = \sin(\pi x/4)$

   answer: $\sum_{n=0}(-1)^n \frac{\pi^{2n+1}}{4^{2n+1}(2n+1)!}x^{2n+1}$