1. **Derivative Formulas**

   (a) Common Derivatives

   i. $\frac{d}{dx}(c) = 0$
   ii. $\frac{d}{dx}(f \pm g) = f' \pm g'$
   iii. $\frac{d}{dx}(x) = 1$
   iv. $\frac{d}{dx}(kx) = k$
   v. Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$
   vi. $\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1} \cdot f'(x)$
   vii. Product Rule: $(f\ g)' = f'g + fg'$
   viii. Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
   ix. Chain Rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$
   x. $\frac{d}{dx}(a^x) = a^x \ln a$
   xi. $\frac{d}{dx}(e^x) = e^x$
   xii. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
   xiii. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$
   xiv. $\frac{d}{dx}\left(e^{f(x)}\right) = f'(x) \cdot e^{f(x)}$
   xv. $\frac{d}{dx}(\ln(f(x))) = \frac{1}{f(x)} \cdot f'(x)$
   xvi. $\frac{d}{dx}(\sin x) = \cos x$
   xvii. $\frac{d}{dx}(\cos x) = -\sin x$
   xviii. $\frac{d}{dx}(\tan x) = \sec^2 x$
   xix. $\frac{d}{dx}(\sec x) = \sec x\ \tan x$
   xx. $\frac{d}{dx}(\csc x) = -\csc x\ \cot x$
   xxi. $\frac{d}{dx}(\cot x) = -\csc^2 x$
   xxii. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$
   xxiii. $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$
   xxiv. $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$
   xxv. $\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$
   xxvi. $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2-1}}$
   xxvii. $\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{x\sqrt{x^2-1}}$

2. **L'Hospitals Rule**

   (a) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, then

   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \quad , g'(x) \neq 0$$

   (b) If $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$, then

   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

   (c) **Indeterminate Form of Type** $0 \cdot \infty$

   Given $\lim_{x\to a} f(x)g(x)$, you do the following to put it in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

   i. Rewrite $f(x)g(x)$ as $\frac{f(x)}{1/g(x)}$
   ii. Rewrite $f(x)g(x)$ as $\frac{g(x)}{1/f(x)}$

   (d) **Indeterminate Form of** 0^0**,** ∞^0**, or** 1^∞

   Given $\lim_{x\to a} f(x)^{g(x)}$, rewrite the limit as

   $$\lim_{x\to a} e^{g(x)\ln(f(x))}$$

then take the limit of the exponent

$$\lim_{x\to a} g(x)\ln(f(x))$$

This should put the limit in the **Indeterminate Form of Type** $0 \cdot \infty$

3. **Integrals**

(a) **Common Integrals**

i. $\int k\ dx = kx + C$

ii. $\int x^n\ dx = \frac{x^{n+1}}{n+1} + C,\quad n \neq -1$

iii. $\int \frac{1}{x}\ dx = \ln|x| + C$

iv. $\int \frac{1}{kx+b}\ dx = \frac{1}{k}\ln|kx+b| + C$

v. $\int e^x\ dx = e^x + C$

vi. $\int e^{kx}\ dx = \frac{1}{k}e^{kx} + C$

vii. $\int e^{kx+b}\ dx = \frac{1}{k}e^{kx+b} + C$

viii. $\int a^x\ dx = \frac{a^x}{\ln a} + C$

ix. $\int a^{kx}\ dx = \frac{1}{k\ln a}a^{kx} + C$

x. $\int a^{kx+b}\ dx = \frac{1}{k\ln a}a^{kx+b} + C$

xi. $\int \ln x\ dx = x\ln x - x + C$

xii. $\int \cos x\ dx = \sin x + C$

xiii. $\int \sin x\ dx = -\cos x + C$

xiv. $\int \sec^2 x\ dx = \tan x + C$

xv. $\int \csc^2 x\ dx = -\cot x + C$

xvi. $\int \sec x \tan x\ dx = \sec x + C$

xvii. $\int \csc x \cot x\ dx = -\csc x + C$

xviii. $\int \tan x\ dx = \ln|\sec x| + C$

xix. $\int \cot x\ dx = \ln|\sin x| + C$

xx. $\int \csc x\ dx = \ln|\csc x - \cot x| + C$

xxi. $\int \sec x\ dx = \ln|\sec x + \tan x| + C$

xxii. $\int \frac{1}{\sqrt{a^2 - x^2}}\ dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

xxiii. $\int \frac{1}{a^2 + x^2}\ dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

xxiv. $\int -\frac{1}{a^2 + x^2}\ dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

xxv. $\int \frac{1}{x\sqrt{x^2 - 1}}\ dx = \sec^{-1}(x) + C$

xxvi. $\int -\frac{1}{x\sqrt{x^2 - 1}}\ dx = \csc^{-1}(x) + C$

4. **Integration Techniques**

(a) u **Substitution**

Given $\int_a^b f(g(x))g'(x)\ dx$,

i. Let $u = g(x)$

ii. Then $du = g'(x)\ dx$

iii. If there are bounds, you must change them using $u = g(b)$ and $u = g(a)$

$$\int_a^b f(g(x))g'(x)\ dx = \int_{g(a)}^{g(b)} f(u)\ du$$

(b) **Integration By Parts**

$$\int u\ dv = uv - \int v\ du$$

Example: $\int x^2 e^{-x}\ dx$

$$u = x^2 \qquad dv = e^{-x}\,dx$$
$$du = 2x\,dx \qquad v = -e^{-x}$$
$$\int x^2 e^{-x}\,dx = -x^2 e^{-x} - \int -2xe^{-x}\,dx$$

You may have to do integration by parts more than once. When trying to figure out what to choose for u, you can follow this guide: LIATE

L Logs

I Inverse Trig Functions

A Algebraic (radicals, rational functions, polynomials)

T Trig Functions ($\sin x$, $\cos x$)

E Exponential Functions

(c) **Products of Trig Functions**

i. $\int \sin^n x \cos^m x\,dx$

A. m is odd (power of $\cos x$ is odd). Factor out one $\cos x$ and place it in front of dx. Rewrite all remaining $\cos x$ as $\sin x$ by using $\cos^2 x = 1 - \sin^2 x$. Then let $u = \sin x$ and $du = \cos x\,dx$

B. n is odd (power of $\sin x$ is odd). Factor out one $\sin x$ and place it in front of dx. Rewrite all remaining $\sin x$ as $\cos x$ by using $\sin^2 x = 1 - \cos^2 x$. Then let $u = \cos x$ and $du = -\sin x\,dx$

C. If n and m are both odd, you can choose either of the previous methods.

D. If n and m are even, use the following trig identities

$$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$$
$$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$
$$\sin(2x) = 2\cos x \sin x$$

ii. $\int \tan^n x \sec^m x\,dx$

A. m is even (power of $\sec x$ is even). Factor out one $\sec^2 x$ and place it in front of dx. Rewrite all remaining $\sec x$ as $\tan x$ by using $\sec^2 x = 1 + \tan^2 x$. Then let $u = \tan x$ and $du = \sec^2 x\,dx$

B. n is odd (power of $\tan x$ is odd). Factor out one $\sec x \tan x$ and place it in front of dx. Rewrite all remaining $\tan x$ as $\sec x$ by using $\tan^2 x = \sec^2 x - 1$. Then let $u = \sec x$ and $du = \sec x \tan x\,dx$

C. If n odd and m is even, you can use either of the previous methods.

D. If n is even and m is odd, the previous methods will not work. You can try to simplify or rewrite the integrals. You may try other methods.

(d) **Partial Fractions**

Use this method when you are integrating $\int \frac{p(x)}{q(x)}\,dx$, the degree of $p(x)$ must be smaller than the degree of $q(x)$. Factor the denominator $q(x)$ into a product of linear and quadratic factors. There are four scenarios.

Unique Linear Factors: If your denominator has unique linear factors

$$\frac{x}{(2x-1)(x-3)} = \frac{A}{2x-1} + \frac{B}{x-3}$$

To solve for A and B, multiply through by the common denominator to get

$$x = A(x-3) + B(2x-1)$$

You can find A by plugging in $x = \frac{1}{2}$. Find B by plugging in $x = 3$.

Repeated Linear Factors: Every power of the linear factor gets its own fraction (up to the highest power).

$$\frac{x}{(x-3)(3x+4)^3} = \frac{A}{x-3} + \frac{B}{3x+4} + \frac{C}{(3x+4)^2} + \frac{D}{(3x+4)^3}$$

Unique Quadratic Factor:

$$\frac{3x-1}{(x+4)(x^2+9)} = \frac{A}{x+4} + \frac{Bx+C}{x^2+9}$$

To solve or A, B, and C, multiply through by the common denominator to get

$$3x - 1 = A(x^2+9) + (Bx+C)(x+4)$$

Repeated Quadratic Factor: Every power of the quadratic factor gets its own fraction (up to the highest power).

$$\frac{2x}{(3x+4)(x^2+9)^3} = \frac{A}{3x+4} + \frac{Bx+C}{x^2+9} + \frac{Dx+E}{(x^2+9)^2} + \frac{Fx+G}{(x^2+9)^3}$$

(e) **Trig Substitution**

If you see	Substitute	Uses the following Identity
$\sqrt{a^2-x^2}$	$x = a\sin(\theta)$	$1-\sin^2(\theta) = \cos^2(\theta)$
$\sqrt{a^2+x^2}$	$x = a\tan(\theta)$	$1+\tan^2(\theta) = \sec^2(\theta)$
$\sqrt{x^2-a^2}$	$x = a\sec(\theta)$	$\sec^2(\theta) - 1 = \tan^2(\theta)$

Example: $\int \frac{x^3}{\sqrt{16-x^2}}\,dx$ Let $x = 4\sin\theta$, $dx = 4\cos\theta\,d\theta$. So $x^3 = 64\sin^3\theta$

$$\int \frac{x^3}{\sqrt{16-x^2}}\,dx = \int \frac{64\sin^3\theta}{\sqrt{16-16\sin^2\theta}} \cdot 4\cos\theta\,d\theta = \int \frac{64\sin^3\theta \cdot 4\cos\theta d\theta}{4\cos\theta} = \int \sin^3\theta\,d\theta$$

Now you have an integral containing powers of trig functions. You can refer to that method to solve the rest of this integral.

$$\int \sin^3\ d\theta = \frac{\cos^3\theta}{3} - \cos\theta + C$$

To get back to x, we need to use a right triangle with the original substitution $x = 4\sin x$.