1. Derivative Formulas

(a) Common Derivatives

i. \( \frac{d}{dx}(c) = 0 \)

ii. \( \frac{d}{dx}(f \pm g) = f' \pm g' \)

iii. \( \frac{d}{dx}(x) = 1 \)

iv. \( \frac{d}{dx}(kx) = k \)

v. Power Rule: \( \frac{d}{dx}(x^n) = nx^{n-1} \)

vi. \( \frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1} \cdot f'(x) \)

vii. Product Rule: \( (f \cdot g)' = f' \cdot g + f \cdot g' \)

viii. Chain Rule: \( \left( f(g(x))' \right) = f'(g(x)) \cdot g'(x) \)

ix. \( \frac{d}{dx}(a^x) = a^x \ln a \)

x. \( \frac{d}{dx}(e^x) = e^x \)

xi. \( \frac{d}{dx}(\ln x) = \frac{1}{x} \)

xii. \( \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a} \)

xiii. \( \frac{d}{dx}(e^{f(x)}) = f'(x) \cdot e^{f(x)} \)

xiv. \( \frac{d}{dx}(\ln(f(x))) = \frac{1}{f(x)} \cdot f'(x) \)

xv. \( \frac{d}{dx}(\sin x) = \cos x \)

xvi. \( \frac{d}{dx}(\cos x) = -\sin x \)

xvii. \( \frac{d}{dx}(\tan x) = \sec^2 x \)

xviii. \( \frac{d}{dx}(\sec x) = \sec x \tan x \)

xix. \( \frac{d}{dx}(\sec x) = -\csc x \cot x \)

xx. \( \frac{d}{dx}(\csc x) = -\csc^2 x \)

xxi. \( \frac{d}{dx}(\cot x) = -\csc^2 x \)

xxii. \( \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}} \)

xxiii. \( \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}} \)

xxiv. \( \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2} \)

xxv. \( \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1 + x^2} \)

xxvi. \( \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x \sqrt{x^2 - 1}} \)

xxvii. \( \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x \sqrt{x^2 - 1}} \)

2. L’Hospitals Rule

(a) If \( \lim_{x \to a} f(x) = 0 \) and \( \lim_{x \to a} g(x) = 0 \), then

\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}, \quad g'(x) \neq 0
\]

(b) If \( \lim_{x \to a} f(x) = \pm \infty \) and \( \lim_{x \to a} g(x) = \pm \infty \), then

\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}
\]

(c) Indeterminate Form of Type 0 \cdot \infty

Given \( \lim_{x \to a} f(x)g(x) \), you do the following to put it in the form \( \frac{0}{\, 0} \) or \( \frac{\infty}{\, \infty} \)

i. Rewrite \( f(x)g(x) \) as \( \frac{f(x)}{1/g(x)} \)

ii. Rewrite \( f(x)g(x) \) as \( \frac{g(x)}{1/f(x)} \)

(d) Indeterminate Form of \( 0^0, \infty^0, \) or \( 1^\infty \)

Given \( \lim_{x \to a} f(x)^g(x) \), rewrite the limit as

\[
\lim_{x \to a} e^{g(x) \ln(f(x))}
\]
then take the limit of the exponent

$$\lim_{x \to a} g(x) \ln(f(x))$$

This should put the limit in the **Indeterminate Form of Type** $0 \cdot \infty$

3. **Integrals**

(a) **Common Integrals**

i. $\int k \, dx = kx + C$

ii. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \ n \neq -1$

iii. $\int \frac{1}{x} \, dx = \ln |x| + C$

iv. $\int \frac{1}{kx + b} \, dx = \frac{1}{k} \ln |kx + b| + C$

v. $\int e^x \, dx = e^x + C$

vi. $\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C$

vii. $\int e^{kx+b} \, dx = \frac{1}{k} e^{kx+b} + C$

viii. $\int a^x \, dx = \frac{a^x}{\ln a} + C$

ix. $\int a^{kx} \, dx = \frac{1}{k \ln a} a^{kx} + C$

x. $\int a^{kx+b} \, dx = \frac{1}{k \ln a} a^{kx+b} + C$

xi. $\int \ln x \, dx = x \ln x - x + C$

xii. $\int \cos x \, dx = \sin x + C$

xiii. $\int \sin x \, dx = -\cos x + C$

xiv. $\int \sec^2 x \, dx = \tan x + C$

xv. $\int \csc^2 x \, dx = -\cot x + C$

xvi. $\int \sec x \tan x \, dx = \sec x + C$

xvii. $\int \csc x \cot x \, dx = -\csc x + C$

xviii. $\int \tan x \, dx = \ln |\sec x| + C$

xix. $\int \cot x \, dx = \ln |\sin x| + C$

xx. $\int \csc x \, dx = \ln |\csc x - \cot x| + C$

xxi. $\int \sec x \, dx = \ln |\sec x + \tan x| + C$

xxii. $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \left( \frac{x}{a} \right) + C$

xxiii. $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$

xxiv. $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \cot^{-1} \left( \frac{x}{a} \right) + C$

xxv. $\int \frac{1}{x\sqrt{x^2 - 1}} \, dx = \sec^{-1} (x) + C$

xxvi. $\int \frac{1}{x\sqrt{x^2 - 1}} \, dx = \csc^{-1} (x) + C$

4. **Integration Techniques**

(a) **u Substitution**

Given $\int_a^b f(g(x))g'(x) \, dx$,

i. Let $u = g(x)$

ii. Then $du = g'(x) \, dx$

iii. If there are bounds, you must change them using $u = g(b)$ and $u = g(a)$

$$\int_a^b f(g(x))g'(x) \, dx = \int_{g(a)}^{g(b)} f(u) \, du$$

(b) **Integration By Parts**

$$\int u \, dv = uv - \int v \, du$$

Example: $\int x^2 e^{-x} \, dx$
\[
\begin{align*}
u &= x^2 & dv &= e^{-x} \, dx \\
du &= 2x \, dx & v &= -e^{-x}
\end{align*}
\]
\[
\int x^2 e^{-x} \, dx = -x^2 e^{-x} - \int -2xe^{-x} \, dx
\]

You may have to do integration by parts more than once. When trying to figure out what to choose for \( u \), you can follow this guide: LIATE

\begin{itemize}
  \item **L** Logs
  \item **I** Inverse Trig Functions
  \item **A** Algebraic (radicals, rational functions, polynomials)
  \item **T** Trig Functions (\( \sin x, \cos x \))
  \item **E** Exponential Functions
\end{itemize}

(c) **Products of Trig Functions**

\begin{enumerate}
\item \( \int \sin^n x \cos^m x \, dx \)
  \begin{enumerate}
  \item **A.** \( m \) is odd (power of \( \cos x \) is odd). Factor out one \( \cos x \) and place it in front of \( dx \). Rewrite all remaining \( \cos x \) as \( \sin x \) by using \( \cos^2 x = 1 - \sin^2 x \). Then let \( u = \sin x \) and \( du = \cos x \, dx \)
  \item **B.** \( n \) is odd (power of \( \sin x \) is odd). Factor out one \( \sin x \) and place it in front of \( dx \). Rewrite all remaining \( \sin x \) as \( \cos x \) by using \( \sin^2 x = 1 - \cos^2 x \). Then let \( u = \cos x \) and \( du = -\sin x \, dx \)
  \item **C.** If \( n \) and \( m \) are both odd, you can choose either of the previous methods.
  \item **D.** If \( n \) and \( m \) are even, use the following trig identities
    \[
    \begin{align*}
    \sin^2 x &= \frac{1}{2}(1 - \cos(2x)) \\
    \cos^2 x &= \frac{1}{2}(1 + \cos(2x)) \\
    \sin(2x) &= 2 \cos x \sin x
    \end{align*}
    \]
\end{enumerate}
\item \( \int \tan^n x \sec^m x \, dx \)
  \begin{enumerate}
  \item **A.** \( m \) is even (power of \( \sec x \) is even). Factor out one \( \sec^2 x \) and place it in front of \( dx \). Rewrite all remaining \( \sec x \) as \( \tan x \) by using \( \sec^2 x = 1 + \tan^2 x \). Then let \( u = \tan x \) and \( du = \sec^2 x \, dx \)
  \item **B.** \( n \) is odd (power of \( \tan x \) is odd). Factor out one \( \sec x \tan x \) and place it in front of \( dx \). Rewrite all remaining \( \tan x \) as \( \sec x \) by using \( \tan^2 x = \sec^2 x - 1 \). Then let \( u = \sec x \) and \( du = \sec x \tan x \, dx \)
  \item **C.** If \( n \) odd and \( m \) is even, you can use either of the previous methods.
  \item **D.** If \( n \) is even and \( m \) is odd, the previous methods will not work. You can try to simplify or rewrite the integrals. You may try other methods.
\end{enumerate}
\end{enumerate}

(d) **Partial Fractions**

Use this method when you are integrating \( \int \frac{p(x)}{q(x)} \, dx \), the degree of \( p(x) \) must be smaller than the degree of \( q(x) \). Factor the denominator \( q(x) \) into a product of linear and quadratic factors. There are four scenarios.
Unique Linear Factors: If your denominator has unique linear factors

\[
\frac{x}{(2x - 1)(x - 3)} = \frac{A}{2x - 1} + \frac{B}{x - 3}
\]

To solve for \(A\) and \(B\), multiply through by the common denominator to get

\[x = A(x - 3) + B(2x - 1)\]

You can find \(A\) by plugging in \(x = \frac{1}{2}\). Find \(B\) by plugging in \(x = 3\).

Repeated Linear Factors: Every power of the linear factor gets its own fraction (up to the highest power).

\[
\frac{x}{(x - 3)(3x + 4)^3} = \frac{A}{x - 3} + \frac{B}{3x + 4} + \frac{C}{(3x + 4)^2} + \frac{D}{(3x + 4)^3}
\]

Unique Quadratic Factor:

\[
\frac{3x - 1}{(x + 4)(x^2 + 9)} = \frac{A}{x + 4} + \frac{Bx + C}{x^2 + 9}
\]

To solve for \(A\), \(B\), and \(C\), multiply through by the common denominator to get

\[3x - 1 = A(x^2 + 9) + (Bx + C)(x + 4)\]

Repeated Quadratic Factor: Every power of the quadratic factor gets its own fraction (up to the highest power).

\[
\frac{2x}{(3x + 4)(x^2 + 9)^3} = \frac{A}{3x + 4} + \frac{Bx + C}{x^2 + 9} + \frac{Dx + E}{(x^2 + 9)^2} + \frac{Fx + G}{(x^2 + 9)^3}
\]

(e) Trig Substitution

<table>
<thead>
<tr>
<th>If you see</th>
<th>Substitute</th>
<th>Uses the following Identity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{a^2 - x^2})</td>
<td>(x = a \sin(\theta))</td>
<td>(1 - \sin^2(\theta) = \cos^2(\theta))</td>
</tr>
<tr>
<td>(\sqrt{a^2 + x^2})</td>
<td>(x = a \tan(\theta))</td>
<td>(1 + \tan^2(\theta) = \sec^2(\theta))</td>
</tr>
<tr>
<td>(\sqrt{x^2 - a^2})</td>
<td>(x = a \sec(\theta))</td>
<td>(\sec^2(\theta) - 1 = \tan^2(\theta))</td>
</tr>
</tbody>
</table>

Example: \(\int \frac{x^3}{\sqrt{16 - x^2}} \, dx\) Let \(x = 4 \sin(\theta)\), \(dx = 4 \cos(\theta) \, d\theta\). So \(x^3 = 64 \sin^3(\theta)\)

\[
\int \frac{x^3}{\sqrt{16 - x^2}} \, dx = \int \frac{64 \sin^3(\theta)}{\sqrt{16 - 16 \sin^2(\theta)}} \cdot 4 \cos(\theta) \, d\theta = \int \frac{64 \sin^3(\theta) \cdot 4 \cos(\theta) \, d\theta}{4 \cos(\theta)} = \int \sin^3(\theta) \, d\theta
\]

Now you have an integral containing powers of trig functions. You can refer to that method to solve the rest of this integral.

\[
\int \sin^3(\theta) \, d\theta = \frac{\cos^3(\theta)}{3} - \cos(\theta) + C
\]

To get back to \(x\), we need to use a right triangle with the original substitution \(x = 4 \sin(\theta)\).