Math 230

Calculus II

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Formula 1: Stategy for Testing Series

Test	Theorem	Comments
Divergence Test	Given the series $\sum a_n$, if $\lim\limits_{n\to\infty} a_n \neq 0$, the series diverges.	Can only be used to test for divergence. It can never be used to test for convergence.
Geometric Series	A geometric seres $\sum_{n=0} ar^n$ converges to $\frac{a}{1-r}$ if $-1 < r < 1$.	If you notice that all the factors are either constants or powers of n, then try writing it as a geometric series $\sum ar^n$.
Integral Test	Given the series $\sum a_n$, if you can write $a_n = f(n)$ and $\int_1^\infty f(x)\, dx$ converges, then so does $\sum a_n$. If $\displaystyle\int_1^\infty f(x)\, dx$ diverges, so does $\sum a_n$	The function f should be easily integrated using one of our integration techniques from previous chapters. This test should not be used when a_n has factorials.
P-Test	Given the series $\sum \dfrac{1}{n^p}$, if $p > 1$ then $\sum \frac{1}{n^p}$ converges. Otherwise it diverges.	Useful when using the comparsion tests. You can compare a rational sequence to a p series.
Direct Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series such that $0 \leq a_n \leq b_n$. If $\sum b_n$ converges, so does $\sum a_n$. If $\sum a_n$ diverges, so does $\sum b_n$.	Anticipate the convergence. This allows you to choose an appropriate comparison. You should know the convergence of this series.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series where $a_n \geq 0$ and $b_n \geq 0$. If $\lim_{n\to\infty} \dfrac{a_n}{b_n} = L$, where L is a positive finite number, then either both series converge or they both diverge.	Easier to use than the Direct Comparison Test since getting the required inequalities to work can be time consuming. You should know the convergence of one of the series.
Alternating Series Test	Given the series $\sum(-1)^n b_n$, if $b_n > 0$, decreasing, and $b_n \to 0$, then the series converges.	Sometimes it's easier to check $\sum \lvert(-1)^n b_n\rvert = \sum b_n$. If this converges, then $\sum(-1)^n b_n$ converges absolutely.
Ratio Test	Let $\sum a_n$ be a series. Let $L = \lim_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert$. If $L < 1$, the series converges absolutely. If $L > 1$, the series diverges. If $L = 1$, the test is inconclusive.	Use this test when you have exponential functions or factorials.
Root Test	Let $\sum a_n$ be a series. Let $L = \lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert}$. If $L < 1$, the series converges absolutely. If $L > 1$, the series diverges. If $L = 1$, the test is inconclusive.	Use this test when $a_n = (b_n)^n$.
Absolute Convergence	If the series $\sum a_n$ converges absolutely, then it converges.	A series $\sum a_n$ converges absolutely, when $\sum \lvert a_n\rvert$ converges.
Conditional Convergence	If the series $\sum a_n$ converges but $\sum \lvert a_n\rvert$ diverges, then the series $\sum a_n$ converges conditionally.	Sometimes you're asked to determine if a series is conditional convergent. You must show convergence for $\sum a_n$ and divergence for $\sum \lvert a_n\rvert$.