Math 230 **Final Examination** Name: ______________________

May 13, 2008

No books, notes, or graphing calculators are allowed on this test. Your instructor will provide you with scratch paper, if you need it. Be sure that all of your work is shown and that it is well organized and legible. Each question is worth 10 points.

(1) The region bounded by $y = 2\sqrt{x}$ and the lines $y = 1$ and $x = 4$ is revolved about the x-axis. Set up **but do not evaluate** an integral to find the volume of this solid.

(2) Differentiate the following, showing all steps: $g(t) = e^{2t}\sin^{-1} t$

(3) Differentiate $y = \ln\left(\dfrac{e^x\sqrt{x^2+1}}{(x+1)^3}\right)$ [Hint: use the properties of ln.]

(4) Give the precise definitions of
(a) $\ln(x)$ $(x > 0)$

(b) $\exp(x) = e^x$

(5) Find the length of the curve $y = x^{3/2}$, $0 \leq x \leq 4$.

(6) Let $f(x)$ be a function defined for $x \geq 0$ whose derivative is $f'(x) = \dfrac{6}{x+2}$.
(a) Explain why $f(x)$ is one–to–one for $x \geq 0$.

Since $f(x)$ is one–to–one, there is an inverse function $g(x) = f^{-1}(x)$.
(b) If $f(1) = 5$, compute $g'(5)$.

(7) Find $\lim_{x\to 1} \frac{\frac{\pi}{4} - \tan^{-1} x}{x^2 - 1}$

(8) Find $\int \frac{\sqrt{x^2-1}}{x^3}\,dx$

(9) Compute $\int_0^{\pi/4} \tan^2 x \sec^4 x \, dx$

(10) Find $\int \frac{x\,dx}{x^2 + 3x + 2}$

(11) Find $\int e^x \sin(x)\, dx$

(12) Evaluate $\int_1^\infty \frac{2x}{1+x^4}\, dx$

(13) Define the sequence $\{a_n\}$ recursively by $a_{n+1} = 1 + \frac{1}{a_n}$ and $a_1 = 1$. Find a_2, a_3, a_4 and a_5 (write them as exact fractions, please).

(14) Find the limit of the following **sequence** as $n \to \infty$: $a_n = \left(1 + \frac{3}{2n}\right)^n$

(15) Short answer. No partial credit. No work need be shown. In each case answer COVERGES, DIVERGES, or INCONCLUSIVE.

(a) If a_n are nonnegative, decreasing to 0, then $\sum_{n=1}^{\infty} a_n$ __________

(b) If a_n are nonnegative, decreasing to 0, then $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ __________

(c) If $a_n \geq \frac{1}{n}$, then $\sum_{n=1}^{\infty} a_n$ __________

(d) If $\frac{1}{n^2} \geq a_n \geq 0$, then $\sum_{n=1}^{\infty} a_n$ __________

(e) If $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$ then $\sum_{n=1}^{\infty} a_n$ __________

(16) Test the following series for convergence: $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$

(17) Find the exact sum of the series $\sum_{n=1}^{\infty} \frac{4+(-3)^n}{5^{n-1}}$

(18) Find the radius and interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n^2\,5^n}$

(19) (a) Use the expansion $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ to find a power series representation of $\dfrac{1}{1+x^3}$.

(b) Use your answer to express $\displaystyle\int_0^{1/2} \frac{1}{1+x^3}\,dx$ as a power series.

(20) (a) Write down the Maclaurin series expansion of e^x.

(b) Using (a), find the Maclaurin series expansion of e^{-x}.

(c) Estimate the magnitude of the error involved in using the sum of the first four terms of the series in (b) to approximate e^{-1}.