Math 230 Final (Fall 2016) Section ( )

NAME (Print) : _____

ZID : _____

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	

1 (10 points). Set up, but do not evaluate, an integral for computing the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = x$, $x = 4$, and $x = 9$ about the line $y = -2$. State which method you use to get your integral, and **sketch** a typical washer or shell, as appropriate.

2 (20 points). Find dy/dx for the following functions, showing all steps.

a) $y = (\tan^{-1} x + e^x) \cdot \sin^{-1}(2x)$

b) $y = x^{2^x}$

3 (10 points). Show that the function $f(x) = e^x + x$ is one-to-one on $(-\infty, \infty)$, then find the slope of the tangent line to the graph of $y = f^{-1}(x)$ at the point where $x = 1$.

4 (20 points). Evaluate the following limits.

a) $\lim_{x \to 0} \dfrac{x^4 + 7x}{e^{2x} - 1}$

b) $\lim_{x \to \infty} x^{\frac{1}{x}}$

5 (20 points). Evaluate the following integrals.

a) $\int_0^1 x^3\sqrt{1-x^2}\,dx$

b) $\int x^2 e^x\,dx$

c) $\int \frac{1-3x^2}{x^2(x^2+1)}\,dx$

6 (10 points). Find the Taylor polynomial of degree $n = 3$ for $f(x) = e^{-x}$ centered at $a = 0$. Use Taylor's Inequality to estimate the accuracy of the appoximation $f(x) \approx T_3(x)$ when x lies in the interval $[0, 1]$.

7 (10 points). Compute the improper integral $\int_2^\infty \frac{1}{x^2+4}\,dx$.

8 (20 points). Suppose $\sum_{n=0}^{\infty} c_n(x-1)^n$ is a power series with the radius of convergence $R = 6$. What, if anything, can you say about the convergence or divergence of the following series? Be sure to state your reasoning.

a) $\sum_{n=0}^{\infty} c_n 7^n$

b) $\sum_{n=0}^{\infty} c_n(-1)^n 6^n$

c) $\sum_{n=0}^{\infty} c_n$

d) $\sum_{n=0}^{\infty} n c_n 3^{n-1}$

9 (10 points). Does the **sequence** $a_n = (-1)^{2n+1}\cos\left(\frac{2016}{(2n+1)!}\right)$ converge? Explain your answer, and find the limit, if it exists. Does the **series** $\sum_{n=0}^{\infty}(-1)^{2n+1}\cos\left(\frac{2016}{(2n+1)!}\right)$ converge? Explain your answer.

10 (20 points). Determine whether the following series converge absolutely, converge conditionally, or diverge. Be sure to state your reasoning.

a) $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{2n^3+2}$

b) $\sum_{n=0}^{\infty}\frac{(-1)^n (n!)^2}{(2n)!}$

11 (10 points). Find the sum of the following power series: $\sum_{n=3}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{3^{n-1}}$

12 (10 points). Find power series representation for $f(x) = \ln(1+x^2)$

13 (10 points). Find all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n-1)^n x^{n-1}}$ converges.

14 (10 points). Use the Trapezoidal Rule to approximate $\int_2^6 e^{x^2} dx$ using $n = 4$ subintervals.