

Name _____

Math 230
Final Exam
Fall 2014

Some useful reduction formulas:

1. $\int \sin^n x \, dx = \frac{-1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$
2. $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$
3. $\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$
4. $\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx$

1. Let R be the region bounded by $y = \sqrt{1 + x^2}$, $y = 0$, $x = 0$ and $x = 2$.

(a) Find the volume of the solid obtained when R is revolved around the x -axis.

(b) Find the volume of the solid obtained when R is revolved around the y -axis.

(c) Write an integral for the volume of the solid obtained when R is revolved around the line $y = -1$. Use disks/washers. DO NOT EVALUATE.

2. Evaluate $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$.

3. Evaluate $\lim_{x \rightarrow \infty} \frac{\ln(x^2 + 5)}{\ln(x + 3)}$.

4. Evaluate $\lim_{x \rightarrow \pi/4} (4x - \pi) \sec(2x)$.

5. Find the derivative of $\tan^{-1}(e^{2x}) + \ln(\sin^{-1} x)$

6. Evaluate $\int \sin^4(2x) \cos^3(2x) dx$

7. Evaluate $\int \tan^{-1} x dx$

8. Evaluate $\int \sqrt{9+x^2} dx$

9. Evaluate $\int \frac{(x+2)dx}{(x+1)(x+3)}$

10. Evaluate $\int_0^2 \frac{3x+5}{x^2+4} dx$

11. Evaluate $\int_0^{\infty} \frac{e^x dx}{1 + e^{2x}}$

12. Determine whether the series $\sum_{n=0}^{\infty} 5 \cdot \frac{(-1)^n \cdot 2^n}{3^{n+1}}$ converges or diverges. If it converges, find its sum.

13. Use the integral test to determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ converges or diverges.

14. Is the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n} \cdot 2^n}$ absolutely convergent, conditionally convergent, or divergent?
Justify your answer.

15. Determine whether the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ converges or diverges. Justify your answer.

16. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} (-1)^n \frac{n \cdot x^n}{3^n}$

17. Find the Taylor polynomial of degree $n = 3$ for the function $f(x) = \sqrt{x+4}$ centered at $a = 0$.

18. Use the Maclaurin series for $\sin x$ to write a power series for $\int_0^x \frac{\sin t}{t} dt$. What will be its radius of convergence?