1. (*20 points*) Determine the area of the bounded region given by the curves $y = 3x^2 - 15$ and $y = 18x + 6$.

2. (*15 points*) The integral $\int_0^1 \sqrt{1 + e^{2x}} \ dx$ represents the arc-length along the graph of e^x from the point (0, 1) to the point (1, e).

(a) Use the mid-point rule with $n = 2$ to approximate the integral.

(b) The straight-line segment from (0, 1) to (1, e) has length $\sqrt{1 + (e - 1)^2}$. Is the approximation greater or smaller than this value? Are the values reasonably close?

3. (*15 points*) Half of a solid spherical ball of gold of radius 1 rests on a table with the flat side down. It is cut into two pieces along a plane parallel to the table at height $\frac{1}{3}$. One of the two pieces has two flat sides, the other has one flat side. Which is the heavier of the two pieces?

4. (*10 points*) Setup and simplify as far as possible, **but do not evaluate**, the integral that gives the length of the portion of the ellipse $\frac{x^2}{4} + y^2 = 1$ in the first quadrant from (0, 1) to (2, 0).

5. (*10 points*) Determine the simplest form of the formula for the surface area of a cone when the height of the cone is assumed equal to h and the radius of its opening is r.

Hint: Revolve the straight line $y = m\,x$ about the x-axis to generate the cone.

6. (*15 points*) Determine if the following limits exist and if so calculate their value.

(a) $\lim_{x \to 1} \frac{5\,x^4 - 4\,x^2 - 1}{10 - x - 9\,x^3}$

(b) $\lim_{x \to 0^+} x \ln x$

(c) $\lim_{x \to \infty} \left(\sqrt{x^2 + x} - x \right)$

7. (*30 points*) Calculate the following integrals.

(a) $\int x \sin(x)\, dx$

(b) $\int \cos^3(x)\, dx$

(c) $\int \frac{4\,x^4}{x^3 - x^2 + x - 1}\, dx = \int \frac{4\,x^4}{(x-1)\left(x^2 + 1\right)}\, dx$

8. (*10 points*) Calculate the following integrals.

(a) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\, dx$

(b) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\, dx$

9. (*10 points*) Determine if the following integrals converge, and if so calculate the value.

(a) $\int_{-1/5}^{4} \frac{2}{x^3} \, dx$

(b) $\int_{0}^{\infty} x \, e^{-x^2} \, dx$

10. (*10 points*) Let $f(x) = \tan(x)$.

   (a) Determine the third degree Taylor polynomial $P_3(x)$ of $f(x)$ with $a = 0$.

   (b) What is the actual error $P_3\left(\frac{\pi}{4}\right) - f\left(\frac{\pi}{4}\right)$ when P_3 is used in place of f?

11. (*15 points*) Let $f(x) = \frac{1}{2+3x}$.

(a) Determine the power series representation of $f(x)$ in the form $\sum_{m=0}^{\infty} a_n x^n$.

(b) Determine the interval of convergence of the power series and the radius of convergence.

12. (*15 points*) Consider the power series $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$

   (a) Determine the interval of convergence of $f(x)$.

   (b) Use the power series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$ to derive the power series of $g(x) = \frac{1}{1+x^2}$.

   (c) Which mathematical operation turns the power series of $g(x)$ into $f(x)$?

   (d) Which function $h(x)$ is produced when this mathematical operation is applied to $g(x)$?

   (e) What is the domain of $h(x)$, and is it different from the domain of $f(x)$?

13. (*25 points*) Answer the following questions. Indicate clearly any test you employ.

(a) Determine if $\sum_{n=0}^{\infty} (-1)^n \cdot \frac{e^n}{3^n}$ converges or diverges. If it converges, then calculate its value.

(b) Determine if $\sum_{n=1}^{\infty} \frac{\sqrt{2n^3-1}}{n^3+5}$ converges or diverges.

(c) Determine if $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ converges or diverges.

(d) Determine if $\sum_{n=1}^{\infty} (-1)^{n+1} (e^{1/n} - 1)$ converges or diverges.

(e) Determine if $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ converges or diverges.