MATH 230
FINAL EXAM REVIEW GUIDE

1. Sketch the region enclosed the the given curves and find its area. (Section 5.1)
   a) $y = \cos(x)$ and $y = 2 - \cos(x)$ for $0 \leq x \leq 2\pi$
   A: 4π

   b) $y = x^4$ and $y = 2 - |x|$
   A: $13/5$

2. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer. (Section 5.2)
   a) $y = 6 - x^2$, $y = 2$; about the x-axis
   A: $\frac{384}{5}\pi$

   b) $y = x^2$, $x = y^2$; about $y = 1$
   A: $\frac{11}{30}\pi$

3. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves around the given line. (Section 5.3)

   a) $y = x^3$, $y = 0$, $x = 1$, and $x = 2$ about the y-axis.
   A: $\frac{62}{5}\pi$

   b) $y = 4x - x^2$, $y = 3$ about $x = -1$.
   A: 8π

4. Find the exact area of the surface obtained by rotating the curve $y = \sqrt{5 - x}$ for $3 \leq x \leq 5$ about the x-axis. (Section 8.2)
   A: $\frac{13}{3}\pi$

5. Find $(f^{-1})'(a)$ for (Section 6.1):
   a) $f(x) = 3x^3 + 4x^2 + 6x + 5$, $a = 5$
   A: $(f^{-1})'(5) = \frac{1}{6}$

   b) $f(x) = x^3 + 3\sin(x) + 2\cos(x)$, $a = 2$
   A: $(f^{-1})'(2) = \frac{1}{3}$

6. Differentiate the function. (Sections 6.2*, 6.3*, 6.4*, 6.6)
   a) $f(x) = x^2 e^{-1/x}$
   A: $f'(x) = e^{-1/x} + 2xe^{-1/x}$

   b) $f(x) = \ln(\sin^2(x))$
   $f'(x) = 2\cot(x)$

   c) $f(x) = x\sin(2^x)$
   A: $f'(x) = x2^x \ln(2)\cos(2^x) + \sin(2^x)$

   d) $f(x) = x^x$
   A: $f'(x) = x^x[1 + \ln(x)]$

   e) $f(x) = (\tan^{-1}(x))^2$
   A: $f'(x) = \frac{2\tan^{-1}(x)}{1+x^2}$

   f) $f(x) = \cos^{-1}(\sin^{-1}(x))$
   A: $f'(x) = \frac{-1}{\sqrt{1-(\sin^{-1}(x))^2}\sqrt{1-x^2}}$

7. Evaluate the integral. (Section 6.2*, 6.3*, 6.4*, 6.6)
   a) $\int \frac{dx}{x\ln(x)} dx$
   A: $\ln(\ln(x)) + C$

   b) $\int \frac{\cos(\ln(t))}{t} dt$
   A: $\sin(\ln(t)) + C$

   c) $\int x2^{x^2} dx$
   A: $\frac{2^{x^2}}{2\ln(2)} + C$

   d) $\int_0^{1/2} \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$
   A: $\frac{\pi^2}{72}$

8. Evaluate the limits. (Section 6.8)
   a) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$
   A: $\frac{1}{2}$

   b) $\lim_{x\to 1^+} x^{1/(1-x)}$
   A: e^{-1}

   c) $\lim_{x\to\infty} \frac{\ln(\sqrt{x})}{x^2}$
   A: 0

9. Evaluate the integral. (Section 7.1, 7.2, 7.3, 7.4)

a) $\int te^{-3t}dt$

A: $\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C$

b) $\int (x^2 + 2x)\cos(x)dx$

A: $(x^2 + 2x)\sin(x) + (2x + 2)\cos(x) - 2\sin(x) + C$

c) $\int_0^{\pi/2} \sin^7(\theta)\cos^5(\theta)d\theta$

A: $1/120$

d) $\int \tan(x)\sec^3(x)dx$

A: $1/3\sec^3(x) + C$

e) $\int \frac{x^2}{\sqrt{9 - x^2}}dx$

A: $9/2\sin^{-1}(x/3) - 1/2x\sqrt{9 - x^2} + C$

f) $\int \frac{\sqrt{x^2 - 1}}{x^4}dx$

A: $\frac{(x^2-1)^{3/2}}{3x^3} + C$

g) $\int_0^{1/2} x\sqrt{1 - 4x^2}dx$

A: $1/12$

h) $\int \frac{5x + 1}{(2x + 1)(x - 1)}dx$

A: $1/2\ln|2x + 1| + 2\ln|x - 1| + C$

i) $\int \frac{4x}{x^3 - x^2 - x + 1}\,dx$

A: $\ln|x - 1| - \frac{2}{x - 1} - \ln|x + 1| + C$

10. Write out the partial fraction decomposition for $\frac{x^6}{x^2 - 4}$. Do not determine the numerical values of the coefficients. (Section 7.4)

A: $x^4 + 4x^2 + 16 + \frac{A}{x - 2} + \frac{B}{x + 2}$

11. Evaluate the improper integral. (Section 7.8)

a) $\int_0^\infty \frac{1}{\sqrt[4]{1+x}}dx$

A: Diverges

b) $\int_1^\infty \frac{\ln(x)}{x^2}dx$

A: Converges

12. Let $a_n = \frac{2n}{3n+1}$. (Sections 11.1, 11.2)

a) Determine whether $\{a_n\}$ is convergent.

A: Converges to 2/3

b) Determine whether $\sum_{n=1}^\infty a_n$ is convergent.

A: By the test for divergence, this series diverges.

c) What requirement do you need on the sequence $\{a_n\}$ for the $\sum_{n=1}^\infty a_n$ to converge?

A: $\lim_{n\to\infty} a_n = 0$.

13. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. (Sections 11.3, 11.4, 11.5, 11.6)

a) $\sum_{n=1}^\infty \frac{(-1)^{n+1}}{\sqrt{n+1}}$

A: (CC)

b) $\sum_{n=1}^\infty (-1)^n \frac{n^2}{n^2+n+1}$

A: (D)

c) $\sum_{n=1}^\infty \frac{n^{10}}{(-10)^{n+1}}$

A: (AC)

d) $\sum_{n=1}^\infty \left(\frac{n^2+1}{2n^2+1}\right)^n$

A: (AC)

e) $\sum_{n=2}^\infty \frac{1}{n\sqrt{\ln(n)}}$

A: (D)

f) $\sum_{n=1}^\infty (-1)^n \frac{\sqrt{n}}{n+5}$

A: (CC)

14. Find the radius of convergence and the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$. (Section 11.8)

    A: ROC: 1, IOC: $[1, 3]$

15. Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$.

    A: ROC: 1, IOC: $(-1, 1]$

16. Find a power series representation for $f(x) = \ln(5 - x)$ and determine the radius of convergence. (Section 11.9)

    A: $\ln(5 - x) = \ln(5) - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}$, ROC: 5