Exam 3 NAME:

MATH 230, SPRING 2017

Directions: Show ALL work for full credit (unless otherwise state).

1. (7 points each) State whether the series converges or diverges and give a **short** explanation of your answer (i.e., state the tests you would use).

   (a) $\sum_{n=1}^{\infty}(1.01)^n$

   (b) $\sum_{n=1}^{\infty}\tan^{-1} n$

   (c) $\sum_{n=1}^{\infty}\frac{1}{3n-1}$

   (d) $\sum_{n=2}^{\infty}\frac{(-1)^n}{n}$

2. (5 points) Your friend says, "Since $\lim\limits_{n\to\infty} \frac{1}{n^2} = 0$, we know that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges." Your friend is right that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, but your friend's reasoning is wrong. Give an example where your friend's reasoning will lead him to the wrong conclusion.

3. (12 points) Given the geometric series $\sum\limits_{n=1} a_n = 4 - \frac{8}{3} + \frac{16}{9} - \frac{32}{27} + \frac{64}{81} - ...$

   (a) Find a formula for a_n

   (b) Does $\sum\limits_{n=1}^{\infty} a_n$ converge? If so, to what?

4. (5 points) Given the convergent series $S = \sum\limits_{n=1}^{\infty} a_n$. Suppose the partial sum is $S_n = 1 + \frac{1}{n}$. What is S? What is $\lim\limits_{n\to\infty} a_n$?

5. (10 points each) Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n^2+1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$

6. (10 points) Given the power series $\sum_{n=0}^{\infty} \frac{(x+1)^n}{n4^n}$, find the radius of convergence and the interval of convergence.

7. (20 points) Let $a_n = \frac{6n-3}{9n}$.

(a) Find S_3 for $\sum_{n=1}^{\infty} a_n$.

(b) Does $\sum_{n=1}^{\infty} a_n$ converge? Explain.

(c) Does $\sum_{n=1}^{\infty} (a_n)^n$ converge? Explain.