EXAM II MATH 230, CALCULUS II, SPRING 2017 NAME: SOLUTION

Show all work to receive full credit.

1. (20 points) Evaluate the following limits

(a) $\lim_{x\to\infty} \frac{\ln(2+e^x)}{x}$ $\Rightarrow \frac{\infty}{\infty}$ USE LH

$$\overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{2+e^x}\cdot e^x}{1}$$

$$= \lim_{x\to\infty} \frac{e^x}{2+e^x} \Rightarrow \frac{\infty}{\infty}$$

$$\overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{e^x} = 1$$

$$\boxed{\lim_{x\to\infty} \frac{\ln(2+e^x)}{x} = 1}$$

(b) $\lim_{x\to 0^+} x^x = 0^0$

(1) REWRITE AS $e^{x\ln x}$ AND FIND $\lim_{x\to 0^+} x\ln x$

(2) $\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} \overset{LH}{=} \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} -x = 0$

(3) FINAL: $\boxed{e^0 = 1}$

2. (10 points each) Evaluate the following integrals.

(a) Evaluate $\int_1^2 \frac{\ln x}{x^2}\,dx$ BY PARTS

$u = \ln x \qquad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx, \quad v = \frac{-1}{x}$

$\Rightarrow -\frac{1}{x}\ln x - \int -\frac{1}{x^2}\,dx$

$= -\frac{1}{x}\ln x + \int \frac{1}{x^2}\,dx$

$= -\frac{1}{x}\ln x - \frac{1}{x}\Big|_1^2$

$= \left[-\frac{1}{2}\ln 2 - \frac{1}{2}\right] - \left[-\frac{1}{1}\ln 1 - 1\right] = -\frac{1}{2}\ln 2 + \frac{1}{2}$

(b) Evaluate $\int \sec^4(x)\tan^7(x)\,dx$

$\int \sec^2 x \tan^7 x\,(\sec^2 x\,dx)$

$= \int (1+\tan^2 x)\tan^7 x\,(\sec^2 x\,dx)$

$\Rightarrow$ LET $u = \tan x$, $du = \sec^2 x\,dx$

$\int (1+u^2)\,u^7\,du$

$= \int u^7 + u^9\,du$

$= \frac{1}{8}u^8 + \frac{1}{10}u^{10} + C$

$= \frac{1}{8}\tan^8 x + \frac{1}{10}\tan^{10} x + C$

(c) Evaluate $\int_0^1 \frac{e^{\tan^{-1} x}}{1+x^2}\, dx$

LET $u = \tan^{-1} x$, $du = \frac{1}{1+x^2} dx$. IF $x=1$, $u = \tan^{-1} 1 = \pi/4$

IF $x=0$, $u = \tan^{-1} 0 = 0$

$$\int_0^{\pi/4} e^u du = e^{\pi/4} - e^0$$

$$= \boxed{e^{\pi/4} - 1}$$

(d) Evaluate $\int \frac{-x-12}{x(x+2)(x-3)}\, dx$

(1) $\frac{-x-12}{x(x+2)(x-3)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-3}$

(2) $-x-12 = A(x+2)(x-3) + Bx(x-3) + Cx(x+2)$

(3) SHORT CUT:

$x=0$: $-12 = A(-6) \rightarrow A = 2$

$x=3$: $-15 = C(15) \rightarrow C = -1$

$x=-2$: $-10 = B(10) \rightarrow B = -1$

(4) $\int \frac{2}{x} + \frac{-1}{x+2} + \frac{-1}{x-3}\, dx$

$= 2\ln|x| - \ln|x+2| - \ln|x-3| + C$

(e) Evaluate $\displaystyle\int \frac{8\,dx}{x^2\sqrt{1-x^2}}$

LET $x = \sin\theta$, $dx = \cos\theta\, d\theta$

$$\Rightarrow \int \frac{8\cos\theta\, d\theta}{\sin^2\theta\sqrt{1-\sin^2\theta}}$$

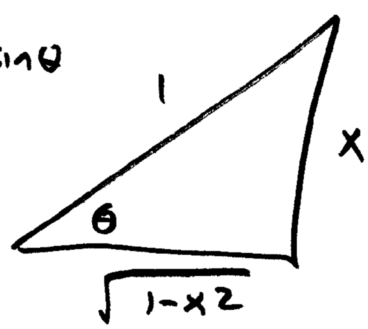
$$= \int \frac{8\cos\theta\, d\theta}{\sin^2\theta\sqrt{\cos^2\theta}}$$

$$= \int \frac{8}{\sin^2\theta}\, d\theta$$

$$= \int 8\csc^2\theta\, d\theta$$

$$= -8\cot\theta + C$$

$\frac{x}{1} = \sin\theta$



$$= -8\cot\theta + C$$

$$= -8\,\frac{\sqrt{1-x^2}}{x} + C$$

3. (16 points) Evaluate the following. No work need be shown.

(a) $\int \cos(3x)\ dx$ $= \frac{1}{3}\sin 3x + C$

(b) $\int \csc x\ dx$ $= \ln|\csc x - \cot x| + C$

(c) $\int \frac{-1}{\sqrt{1-x^2}}\ dx$ $= \cos^{-1}(x) + C$ OR $-\sin^{-1}x + C$

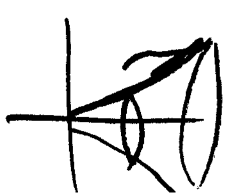
(d) $\int \ln x\ dx$ $= x\ln x - x + C$

4. Let $y = \frac{1}{3}x^3$.

(a) (7 pts) Set up the integral for the length of y over $0 \le x \le 3$. DO NOT EVALUATE.

$$L = \int_0^3 \sqrt{1 + (x^2)^2}\, dx$$

(b) (7 pts) Set up the integral that represents the area of the surface obtained by rotating y about the x-axis.



$$\int_0^3 2\pi f(x)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$= \int^3$ [illegible]