EXAM II - REDO MATH 230, CALCULUS II, SPRING 2017 NAME: SOLUTION

Show all work to receive full credit.

1. (20 points) Evaluate the following limits

(a) $\lim_{x\to 2} \frac{e^x - e^2}{xe^x - e^x - e^2}$

$$\overset{LH}{=} \lim_{x\to 2} \frac{e^x - 0}{(xe^x + e^x) - e^x - 0}$$

SIMPLIFY $= \lim_{x\to 2} \frac{e^x}{xe^x}$

SIMPLIFY $= \lim_{x\to 2} \frac{1}{x} = \frac{1}{2}$

(b) $\lim_{x\to\infty} x^{1/x}$

- REWRITE AS $e^{\frac{1}{x}\ln x}$. FIND $\lim_{x\to\infty} \frac{1}{x}\ln x$

- $\lim_{x\to\infty} \frac{1}{x}\ln x = \lim_{x\to\infty} \frac{\ln x}{x} \overset{LH}{=} \lim_{x\to\infty} \frac{1/x}{1} = 0$

- FINAL: $e^0 = 1$

2. (10 points each) Evaluate the following integrals.

(a) Evaluate $\int_1^e x^4 \ln(x)\, dx$

$$\text{LET } u = \ln x, \quad dv = x^4 dx$$
$$du = \frac{1}{x}dx, \quad V = \frac{1}{5}x^5$$

$$\rightarrow \frac{1}{5}x^5 \ln x - \int \frac{1}{5}x^4 dx$$

$$\rightarrow \frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 \Big|_1^e$$

$$= \left(\frac{1}{5}e^5 \ln e - \frac{1}{25}e^5\right) - \left(\frac{1}{5}\ln 1 - \frac{1}{25}\right)$$

$$= \frac{4}{25}e^5 + \frac{1}{25}$$

(b) Evaluate $\int \sec^4(x) \tan^{40}(x)\, dx$

$$= \int \sec^2(x) \tan^{40}(x) \sec^2(x) dx$$

$$= \int (1 + \tan^2 x) \tan^4(x) \sec^2(x)\, dx$$

$$\text{LET } u = \tan x, \quad du = \sec^2(x)\, dx$$

$$= \int (1+u^2) u^{40} du$$

$$= \int u^{40} + u^{42} du$$

$$= \frac{1}{41}u^{41} + \frac{1}{43}u^{43} + C$$

$$= \frac{1}{41}\tan^{41} x + \frac{1}{43}\tan^{43} x + C$$

(c) Evaluate $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$

LET $u = \sin^{-1} x$, $du = \frac{1}{\sqrt{1-x^2}}\, dx$

$$\rightarrow \int_{x=0}^{x=1} u\, du = \frac{1}{2} u^2 \Big|_{x=0}^{x=1} = \frac{1}{2} (\sin^{-1} x)^2 \Big|_0^1$$

$$\rightarrow \frac{1}{2} (\sin^{-1} 1)^2 - \frac{1}{2} (\sin^{-1} 0)^2$$

$$\rightarrow \frac{1}{2} (\pi/2)^2 - \frac{1}{2} (0)^2 \qquad \boxed{\pi^2/8}$$

(d) Evaluate $\int \frac{4x}{(x-1)^2(x+1)}\, dx$

$$= \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}\, dx$$

$$\frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$\Rightarrow 4x = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

SHORTCUT:

$x = 1$: $4 = B(2) \rightarrow B = 2$

$x = -1$: $-4 = C(4) \rightarrow C = -1$

$$\Rightarrow 4x = Ax^2 - A + Bx + B + Cx^2 - 2Cx + C$$

$$4x = (A+C)x^2 + (B-2C)x + (-A+B+C)$$

$A + C = 0 \rightarrow A = 1$

$B - 2C = 4$

$-A + B + C = 0$

$$\int \frac{1}{x-1} + \frac{2}{(x-1)^2} - \frac{1}{x+1}\, dx = \ln|x-1| - \frac{2}{x-1} - \ln|x+1|$$

(e) Evaluate $\int \frac{1}{\sqrt{25+x^2}}\,dx$

LET $x = 5\tan\theta$, $dx = 5\sec^2\theta\,d\theta$

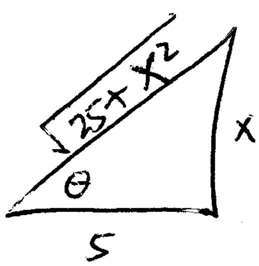
$$\rightarrow \int \frac{5\sec^2\theta\,d\theta}{\sqrt{25+25\tan^2\theta}}$$

$$\rightarrow \int \frac{5\sec^2\theta\,d\theta}{\sqrt{25\sec^2\theta}}$$

$$\rightarrow \int \frac{5\sec^2\theta\,d\theta}{5\sec\theta}$$

$$\rightarrow \int \sec\theta\,d\theta$$

$$\rightarrow \ln|\sec\theta + \tan\theta|$$



$\frac{x}{5} = \tan\theta$

$$\rightarrow \ln\left|\frac{\sqrt{25+x^2}}{5} + \frac{x}{5}\right| + C$$

3. (16 points) Evaluate the following. No work need be shown.

(a) $\int \sin(4x)\ dx$ $= -\frac{1}{4}\cos(4x) + C$

(b) $\int \tan x\ dx$ $= \ln|\sec x| + C$

(c) $\int \frac{-1}{1+x^2}\ dx$ $= \cot^{-1}(x) + C$

(d) $\int \ln x\ dx$ $= x\ln x - x + C$

4. Let $f(x) = 4x^2$, $1 \leq x \leq 3$.

(a) (7 pts) Set up the integral for the length of $f(x)$ over $1 \leq x \leq 3$. DO NOT EVALUATE.

$$L = \int_1^3 \sqrt{1+(8x)^2}\ dx$$

(b) (7 pts) Set up the integral that represents the area of the surface obtained by rotating $f(x)$ about the y-axis.

$$SA = \int_1^3 2\pi x \sqrt{1+(8x)^2}\ dx$$