EXAM 1-B MATH 230, CALCULUS II, SPRING 2017

Name: SOLUTION

Show all work to receive full credit. Scientific calculators only.

1. (8 pts each) Find the derivative of each of the following expressions with respect to x. **You are not required to simplify.**

(a) $f(x) = e^{-3x}\ln(x^2-1)$

$$f'(x) = e^{-3x}\cdot\frac{1}{x^2-1}\cdot 2x + -3e^{-3x}\cdot\ln(x^2-1)$$

(b) $f(x) = \tan^{-1}(3^x)$

$$f'(x) = \frac{1}{1+(3^x)^2}\cdot 3^x\ln 3$$

(c) $y = (\cos x)^x$ using logarithmic differentation.

LOG: $\ln y = \ln((\cos x)^x)$

$$\ln y = x\ln(\cos x)$$

DIFF: $\frac{1}{y}\cdot y' = x\cdot\frac{1}{\cos x}\cdot -\sin x + 1\cdot\ln(\cos x)$

$$y' = y\left(-x\tan x + \ln\cos x\right)$$

$$y' = (\cos x)^x\left(-x\tan x + \ln\cos x\right)$$

2. Integrate the following

(a) (8 pts) $\int \frac{\sin x}{1-\cos x} dx$

(1) LET $u = 1 - \cos x$

(2) $du = \sin x \, dx$

(3) SUBSTITUTE

$$\int \frac{1}{u} du = \ln|u| + C$$

$$= \ln|1 - \cos x| + C$$

(b) (8 points) $\int \frac{1}{x\sqrt{1-(\ln x)^2}} dx$

(1) LET $u = \ln x$

(2) $du = \frac{1}{x} dx$

(3) SUBSTITUTE

$$\int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1}(u) + C$$

$$= \sin^{-1}(\ln x) + C$$

(c) (8 pts) $\int_0^1 y^2 e^{3y^3} dy$

(1) LET $u = 3y^3$

(2) $du = 9y^2 dy$

$\Rightarrow \frac{1}{9} du = y^2 dy$

(3) BOUNDS

IF $y = 1$, $u = 3(1)^3 = 3$

IF $y = 0$, $u = 3(0)^2 = 0$

(4) SUBSTITUTE

$$\int_0^3 \frac{1}{9} e^u du$$

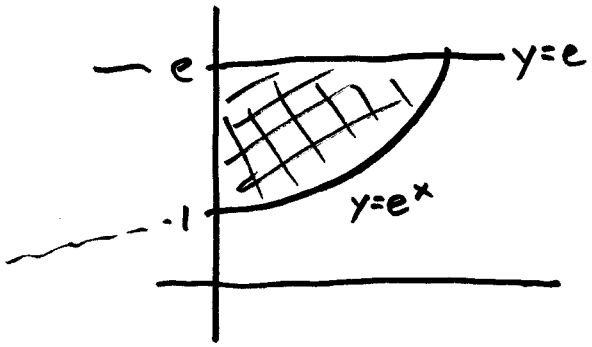
$$= \frac{1}{9} e^u \Big|_0^3$$

$$= \frac{1}{9} e^3 - \frac{1}{9} e^0$$

$$= \frac{1}{9}(e^3 - 1)$$

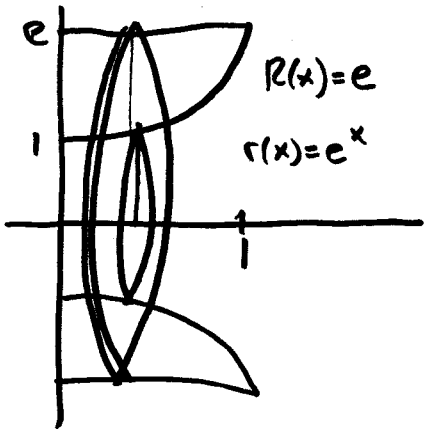
3. Use the curves $y = e^x$, $x = 0$, and $y = e$ to answer the following questions.

(a) (7 pts) Graph the region bounded by the curves.



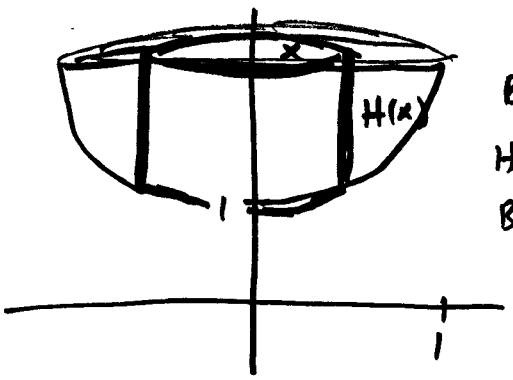
(b) (9 pts each) **Draw the solid and SET UP (do not evaluate)** an integral representing the volume of the solid created when rotating the region about the given lines

i. x-axis using disks/washers



$$V = \int_0^1 \pi(e)^2 - \pi(e^x)^2\,dx$$

ii. y-axis using cylindrical shells



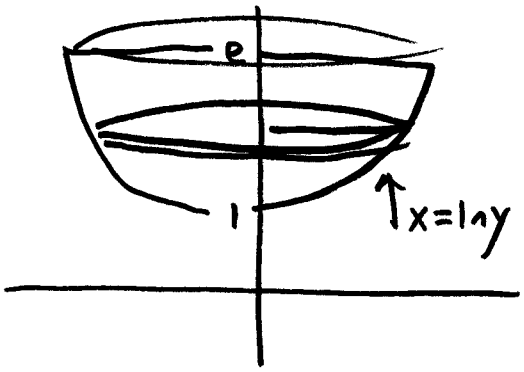
$R(x) = x$

$H(x) = e - e^x$

Bounds: 0 to 1

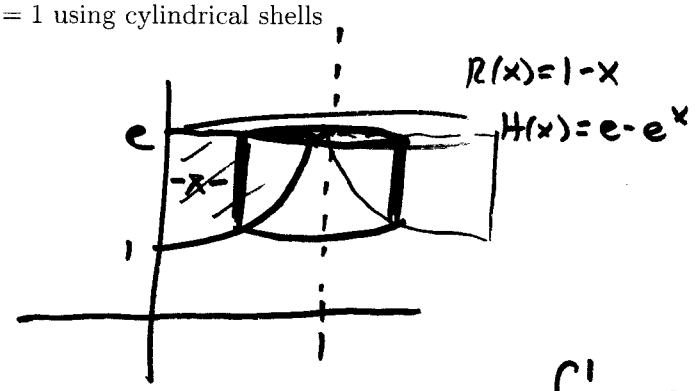
$$\int_0^1 2\pi x(e - e^x)\,dx$$

iii. y-axis using disks/washers



$$\int_1^e \pi (\ln y)^2 \, dy$$

iv. $x = 1$ using cylindrical shells



$$\int_0^1 2\pi(1-x)(e-e^x)\, dx$$

4. (5 points) Circle True or False to the following statements.

(a) (T) / F $\quad \sin^{-1}(\sqrt{2}/2) = \pi/4$

(b) T / (F) $\quad \int \frac{1}{\sqrt{x}}\, dx = \ln|\sqrt{x}| + C$

(c) (T) / F $\quad \lim_{x \to -\infty} e^x = 0$

(d) T / (F) $\quad \lim_{x \to -\infty} \tan^{-1}(x) = \pi/2$

(e) (T) / F $\quad \frac{d}{dx}\left[\cot^{-1} x\right] = -\frac{1}{1+x^2}$