Name:

Math 229
Mock Final Exam

Disclaimer: This mock exam is for practice purposes only. No graphing calulators $\geq$ TI-89 is allowed on this test. Be sure that all of your work is shown and that it is well organized and legible.

This exam's difficulty is on par with a Fall/Spring Final Exam. This also means this final exam is meant for a 1 hour and 50 minute exam. You have 1 hour and 15 minutes. So your exam will be shorter.

Since you were not tested on anti differentiation / integrals, your final exam will have a greater emphasis on this subject.

Good luck!

1. (10pts) True or False. Cirlce your answer.

(a) T F $f(x) = |x|$ is continuous at $x = 0$

(b) T F $f(x) = |x|$ is differentiable at $x = 0$

(c) T F $f(x) = \sqrt{x}$ is differentiable at $x = 0$

(d) T F If a is a critical value of f, then f must have a maximum or minimum at $x = a$.

(e) T F If $f''(a) = 0$, then a must be an inflection point.

2. (24 points) Find the following limits. If the limit is infinite, write ∞ or $-\infty$.

(a) $\lim_{x\to 2} \frac{x^2 - 2x}{x^2 + 4x - 12}$

(b) $\lim_{x\to 0} \frac{\sin(x)}{5x + 8x^2}$

(c) $\lim_{x\to 6^-} \frac{x+6}{x(x-6)}$

(d) $\lim_{x\to 1} \frac{\sqrt{x^2 + 3x} - 2}{x - 1}$

3. (10 points) Let

$$f(x) = \begin{cases} -x^2, & \text{if } x < 0 \\ \cos(x), & \text{if } 0 \leq x < \pi/2 \\ x - \pi/2, & \text{if } x \geq \pi/2 \end{cases}$$

(a) Find $\lim_{x \to 0} f(x)$, if it exists.

(b) Is $f(x)$ continuous at $x = 0$

(c) Is $f(x)$ continuous at $x = \pi/2$

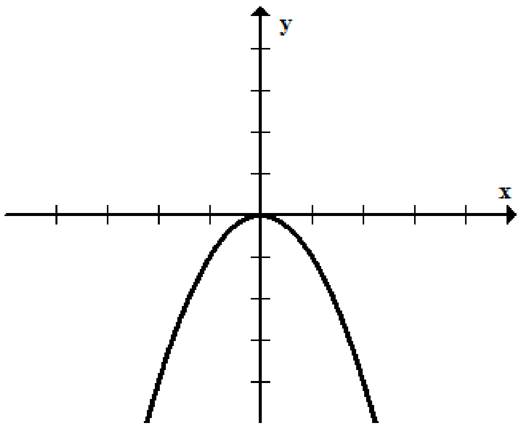
4. (10 points) Let $f(x) = \dfrac{-16}{x}$. Use the limit definition to find $f'(2)$. No credit for any other method.

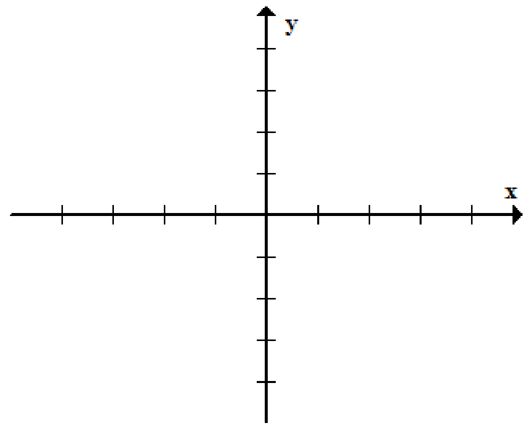
5. (8 points) Find the equation of the tangent line to the graph $f(x) = \cos(x) + x$ at the point $(0, 1)$.

6. (8 points) Let $f(x) = 3x^5 - 5x^3$. Use Newton's Method with thte initial approxiation $x_1 = 1.5$. Find x_2 and x_3.

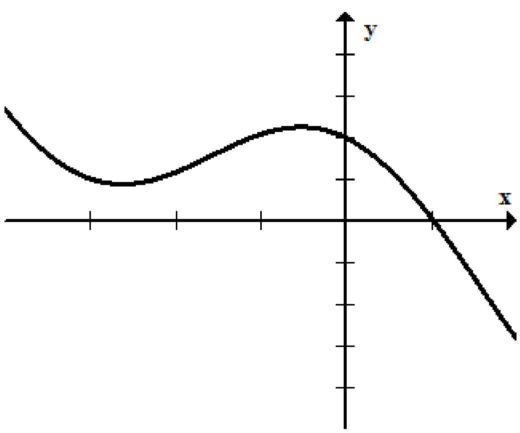
7. (8 points) In each part, the graph of a function is given. Draw a graph of the derivative of this function in the adjacent coordinate system.

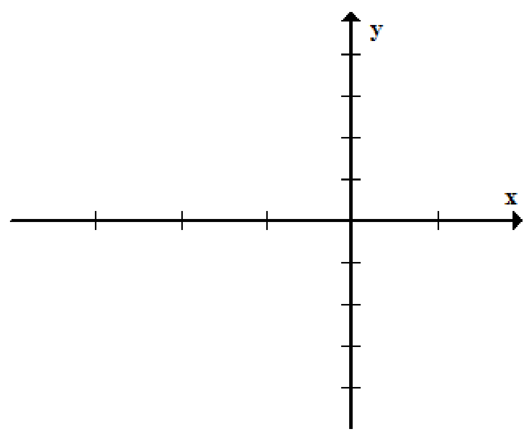
(a)





(b)





8. (24 points) Find the derivative of the following functions. You do not need to simplify.

(a) $f(x) = \sqrt{\sin(x^2)}$

(b) $f(x) = (2x+7)^{10}(5x-3)^4$

(c) $f(x) = \dfrac{2x^6 - 7x + 5}{x^{4/3} + x^{-1/2}}$

(d) $f(x) = \csc^2(x) + x^9$

9. (8 points) Find $\frac{d^2y}{dx^2}$ if $y = x\sec(x)$

10. (6 points) Use implicit differentiation to find $\frac{dy}{dx}$ if $y + x^3 + y^3 = 3.14159$

11. (10 points) Find the critical points of the function $f(x) = \frac{x^2-3}{x-2}$. Determine whether each critical point is a local maximum, minimum, or neither.

12. (6 points) Find the derivative $g'(x)$ of the function $g(x) = \int_5^{x^2} \sqrt{1+t}\, dt$

13. (10 points) Sketch the graph of a funcon satisfying all of the following conditions, labeling all asymptotes, local extrema, and inflection points.

(a) Vertical asymptote at $x = 1$

(b) Horizontal asymptote at $y = -2$

(c) x-intercepts (0,0) and (3,0)

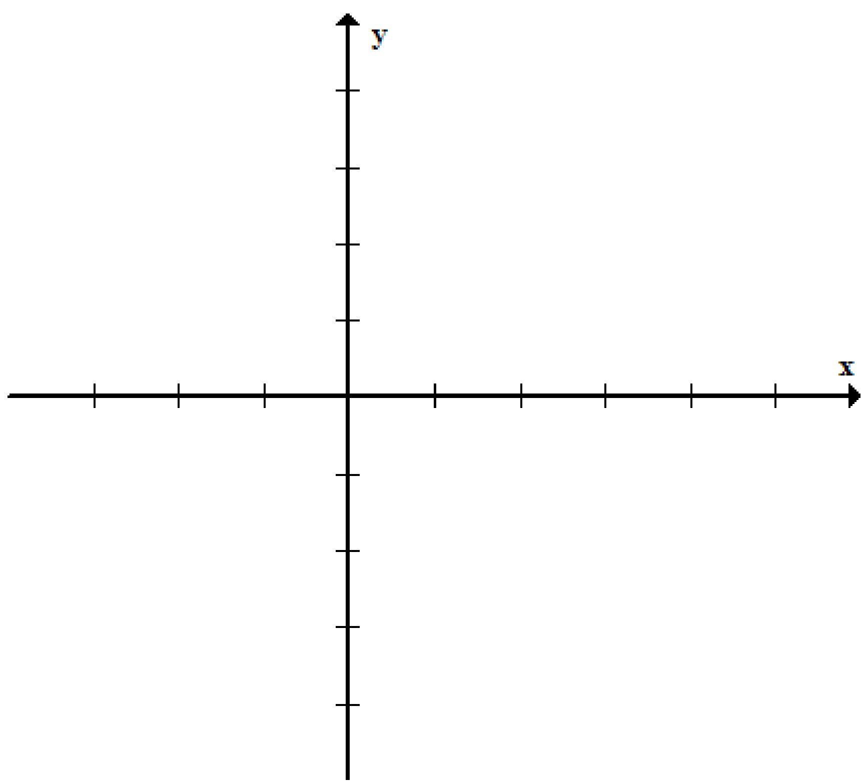
(d) $f(4) = -1$

(e) $f'(x) > 0$ if $x < 3$, $x \neq 1$

(f) $f'(x) < 0$ if $x > 3$

(g) $f''(x) > 0$ if $x < 1$ or $x > 4$

(h) $f''(x) < 0$ if $1 < x < 4$



14. (12 points) A plane is flying horizontally at an altitude of 1 mile and a speed of 500 miles per hour directly over a radar station. Find the rate at which the distance from the plane to the radar station is increasing when the plane is 2 miles from the station.

15. (12 points) Find two numbers x and y whose difference is 2500 and whose product is a minimum.

16. (6 points) Approximate $\int_2^{10} \frac{1}{1+x}\, dx$ using the Riemann Sumn with $n = 4$ rectangles and right-hand endpoints.

17. (28 points) Compute the following integrals.

(a) $\int \frac{x^6 - x^2}{x^4}\, dx$

(b) $\displaystyle\int_0^{\pi/4} \frac{2\tan(x) - 3\sec^3(x)}{\sec(x)}\, dx$

(c) $\displaystyle\int \cos(x) \cdot \sin(\sin(x))\, dx$

(d) $\displaystyle\int_{-1}^{2} |x|\, dx$