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Math 229
Mock Exam I

Disclaimer: This mock exam is for practice purposes only. It may not represent your instructor's exam. Doing well on this exam does not guarantee success on your real exam. It also doesn't guarantee failure. Use this exam to find your strengths and weaknesses and to see how long it takes to do certain problems. One of the main obstacles of any calculus exam is time management.

Good luck!

Calculus Tutoring Center
DU 326

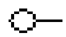
Name:

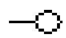
Show all work to receive full credit.

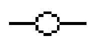
1. Sketch a graph of $f(x)$ that satisfies the following conditions:

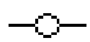
$\lim_{x\to 2^+} f(x) = -4$	$\lim_{x\to 2^-} f(x) = 2$	$f(2) = 3$
$\lim_{x\to -3} f(x) = 1$	f is continuous at $x = -3$	$\lim_{x\to 0^+} f(x) \to \infty$
$\lim_{x\to 0^-} f(x) \to -\infty$	$\lim_{x\to 4} f(x) = 3$	f is not continuous at $x = 4$

Solution

(a) To sketch $\lim_{x\to 2^+} f(x) = -4$, draw  at $(2, -4)$.

(b) To sketch $\lim_{x\to 2^-} f(x) = 2$, draw  at $(2, 2)$.

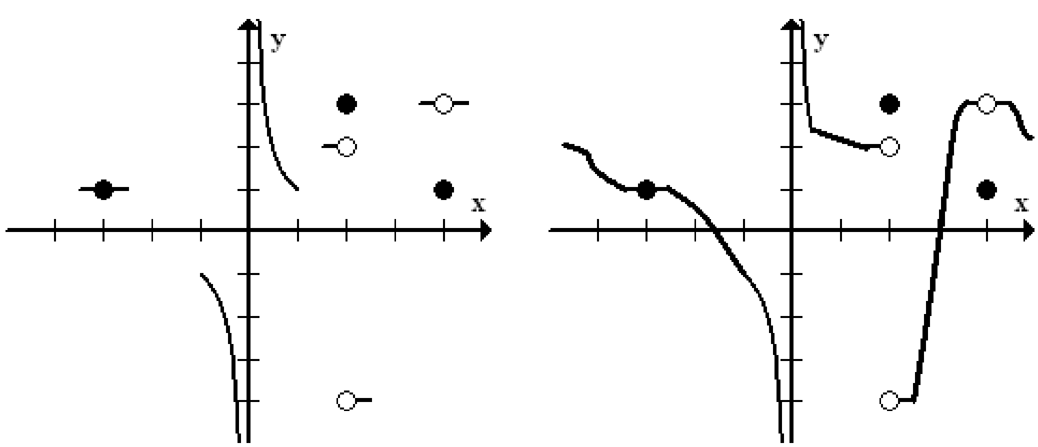
(c) To sketch $\lim_{x\to -3} f(x) = 1$, draw  at $(-3, 1)$.

(d) To sketch $\lim_{x\to 4} f(x) = 3$, draw  at $(4, 3)$.

(e) To sketch $\lim_{x\to 0^-} f(x) = -\infty$, draw a vertical asymptote at $x = 0$

(f) $f(2) = 3$ means plot a point at $(2, 3)$.

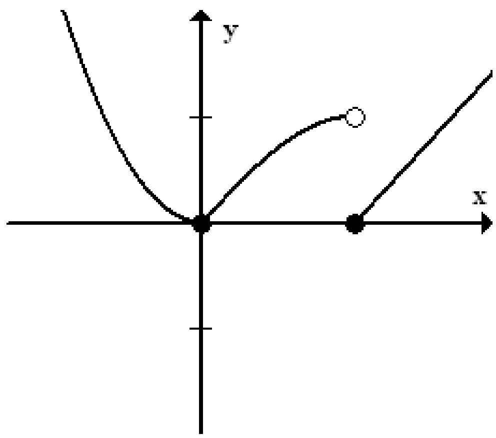
(g) f is continuous at $x = -3$ means the open dot there is now closed.



To finish, connect the dots so you satisfy the conditions.

2. Let $f(x) = \begin{cases} x^2, & x < 0 \\ \sin(x), & 0 \leq x < \pi/2 \\ x - \pi/2, & x \geq \pi/2 \end{cases}$

(a) Graph f. Your graph should be accurate with appropriate tick marks.



(b) Is f continuous at $x = \pi/2$? Explain your answer using limit notation.

What are the conditions to be continuous at $x = \pi/2$?

i. Does $f(\pi/2)$ exist? Yes, $f(\pi/2) = \pi/2 - \pi/2 = 0$.

ii. Does $\lim\limits_{xto\pi/2} f(x)$ exist? No since $\lim\limits_{x\to\pi/2^+} f(x) = \lim\limits_{x\to\pi/2^+} x - \pi/2 = 0$ and $\lim\limits_{x\to\pi/2^-} f(x) = \lim\limits_{x\to\pi/2^-} \sin(x) = 1$. Since the general limit $\lim\limits_{x\to\pi/2} f(x)$ DNE, f is not continuous at $x = \pi/2$.

(c) Is f continuous at $x = 0$? Explain your answer using limit notation.

i. Does $f(0)$ exist? Yes, $f(0) = \sin(0) = 0$.

ii. Does $\lim\limits_{x\to 0} f(x)$ exist? Yes since $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^2 = 0$ and $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \sin(x) = 0$.

iii. Since the general limit $\lim\limits_{x\to 0} f(x) = 0$, and $f(0) = 0$, f is continuous at $x = 0$.

3. Find the limits, if they exist.

(a) $\lim_{x\to 2} \frac{x^2-2x}{3x^2-10x+8}$

i. If you plug in $x=2$, you get $\frac{0}{0}$. So let's try to factor.

ii. $\lim_{x\to 2} \frac{x^2-2x}{3x^2-10x+8} = \lim_{x\to 2} \frac{x(x-2)}{(x-2)(3x-4)} = \lim_{x\to 2} \frac{x}{3x-4} = \frac{2}{2} = 1$

(b) $\lim_{x\to 1} \frac{x\sqrt{5x-1}}{x-3}$

i. Plug in $x=1$.

$$\lim_{x\to 1} \frac{x\sqrt{5x-1}}{x-3} = \frac{1\sqrt{4}}{-2} = -1$$

(c) $\lim_{x\to 4} \frac{3x-12}{|x-4|}$

i. Since you have an absolute value, you need to break with up into a left and right hand limit. Recall that $|x-4|$ is a piece-wise function

$$|x-4| = \begin{cases} x-4, & x \geq 4 \\ -(x-4), & x < 4 \end{cases}$$

ii. Left Hand Limit

$$\lim_{x\to 4^-} \frac{3x-12}{|x-4|} = \lim_{x\to 4^-} \frac{3(x-4)}{-(x-4)} = \lim_{x\to 4^-} \frac{3}{-1} = -3$$

iii. Right Hand Limit

$$\lim_{x\to 4^+} \frac{3x-12}{|x-4|} = \lim_{x\to 4^+} \frac{3(x-4)}{(x-4)} = \lim_{x\to 4^+} \frac{3}{1} = 3$$

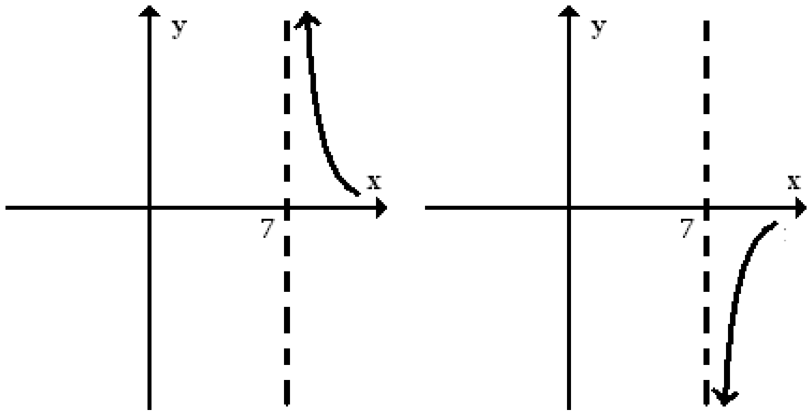
iv. Since $\lim_{x\to 4^-} \frac{3x-12}{|x-4|} \neq \lim_{x\to 4^+} \frac{3x-12}{|x-4|}$, the limit does not exist.

(d) $\lim_{x\to 7^+} \frac{7}{7-x}$

i. If you plug in $x=7$, you get $\frac{7}{0}$ (bad).

ii. Since it's $\frac{nonzero}{zero}$, we have an asympotote.

iii. Since it's a one-sided limit, it will either look like



iv. Try pluggin in values slightly greater than $x = 7$, like $x = 7.1, 7.01, 7.001$. The values get larger in the negative direction.

$$\lim_{x \to 7^+} \frac{7}{7 - x} = -\infty$$

4. Let $f(x) = \begin{cases} \frac{\sqrt{x} - 1}{x - 1}, & x \neq 1 \\ \frac{1}{3}, & x = 1 \end{cases}$

Find $\lim_{x \to 1} f(x)$ and determine if $f(x)$ is continuous at $x = 1$. Show all work and you must use limit notation.

Soluton:

Show f satisfies all the conditions of continuity at $x = 1$.

(a) Does $f(1)$ exist? Yes, and $f(1) = \frac{1}{3}$.

(b) Does $\lim_{x \to 1} f(x)$ exist?

$$\begin{aligned}\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1} &= \lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}\\ &= \lim_{x\to 1}\frac{x+\sqrt{x}-\sqrt{x}-1}{(x-1)(\sqrt{x}+1)}\\ &= \lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{x}+1)}\\ &= \lim_{x\to 1}\frac{1}{\sqrt{x}+1}\\ &= \frac{1}{2}\end{aligned}$$

(c) Since $\lim_{x\to 1} f(x) \neq f(1)$, f is not continuous at $x = 1$.

5. (a) State the Intermediate Value Theorem

Suppose f is continuous on a closed interval $[a, b]$ and let N be any real number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

(b) Use the Intermediate Value Theorem to show the equation $\sqrt{x} + x = 1$ has a solution in the interval $(0, 1)$.

i. Let $f(x) = \sqrt{x} + 1$.

ii. $f(x)$ is continuous on $[0, 1]$.

iii. $f(0) = \sqrt{0} + 0 = 0$

iv. $f(1) = \sqrt{1} + 1 = 2$

v. Since $N = 1$ and $f(0) < N < f(1)$, there exists a number$c$ between $(0, 1)$ such that $f(c) = 1$.

6. Let $f(x) = \dfrac{3}{x-1}$.

(a) Find a formula for $f'(x)$ by using the limit definition of a derivative.

i. Begin by writing

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \quad \text{or} \quad \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

It's your choice. As I'm used to the first one, I'll do that.

$$\begin{aligned}
f'(a) &= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \\
&= \lim_{h\to 0} \frac{\frac{3}{(a+h)-1} - \frac{3}{a-1}}{h} \\
&= \lim_{h\to 0} \frac{\frac{3}{(a+h)-1} - \frac{3}{a-1}}{h} \cdot \frac{(a+h-1)(a-1)}{(a+h-1)(a-1)} \\
&= \lim_{h\to 0} \frac{3(a-1)-3(a+h-1)}{h(a+h-1)(a-1)} \\
&= \lim_{h\to 0} \frac{3a-3-3a-3h+3}{h(a+h-1)(a-1)} \\
&= \lim_{h\to 0} \frac{-3h}{h(a+h-1)(a-1)} \\
&= \lim_{h\to 0} \frac{-3}{(a+h-1)(a-1)} \\
&= \frac{-3}{(a-1)^2}
\end{aligned}$$

(b) Find the equation of the tangent line to $f(x) = \dfrac{3}{x-1}$ at $x = 4$

Using 6(a), the slope of the tangent line at $x = 4$ is $f'(4) = \dfrac{-3}{(4-1)^2} = -\dfrac{1}{3}$

The tangent line is

$$\begin{aligned}
y - 1 &= -\frac{1}{3}(x-4) \\
y &= -\frac{1}{3}x + \frac{7}{3}
\end{aligned}$$