

1. (20 points) Let $f(x) = 4x^5 - 25x^4 + 40x^3 = x^3(4x^2 - 25x + 40)$.

(a) Find all **local** extrema for f .

(b) Determine whether f has an **absolute** maximum and whether it has an **absolute** minimum on the interval $[0, 4)$, and if so, where those absolute extrema are attained.

2. (20 points)

(a) What does the statement “ $x = a$ is a vertical asymptote of the graph of f ” mean?

(b) Is the line $x = 0$ a vertical asymptote of the graph of f for $f(x) = \frac{\sin x}{x}$? Support your answer.

(c) Find all vertical asymptotes of the graph of $y = \frac{1}{x^{5/3} - 9x}$.

3. (20 points) Let $f(x) = x^5 + x^3 + x + 2$

(a) Argue carefully that f is one-to-one. (Name the theorem that supports your argument.)

(b) Argue carefully that the function f has a zero in the interval $[-1, 0]$. (Name the theorem that supports your argument.)

(c) If Newton's Method, beginning with $x_1 = -1$, is used to approximate the zero of f , what is x_2 ?

(d) Determine the concavity of f on $[-1, 0]$. Use this to sketch a rough graph of f on $[-1, 0]$ and show on your graph how Newton's Method constructed your x_2 from your x_1 .

4. (20 points) You are designing a (right, circular) cylindrical container with a capacity $5,000 \text{ cm}^3$. The lateral surface of the container is to be made of thin sheet tin, while the circular top and bottom of the container must be made out of a sturdier tin that is 5 times as thick. What are the dimensions of the container that weighs the least? (Give both the exact dimensions, and their approximations to the nearest one hundredth of a centimeter.)

5. (20 points) Compute the following limits. Show enough work to definitively justify your answer.

(a) $\lim_{x \rightarrow -\infty} \frac{6x^3 - 5x^2}{3x^2 - 2}$

(b) $\lim_{x \rightarrow -\infty} \frac{4x^3 - 5}{\sqrt{10x^6 - 7x^2}}$

(c) $\lim_{x \rightarrow \infty} (\sqrt{9x^2 - 7x + 5} - 3x)$

6. (20 points) Let $f(x) = x^2 - 48x^{2/3}$. Do a full first and second derivative analysis of f , then sketch its graph. Make sure to identify explicitly and graph correctly any vertical and horizontal **tangents**.