MATH 229, Exam III NAME______________________________

1. (9 points) Carefully state the following theorems, making sure you have the hypotheses correct.
   - The Extreme Value Theorem
   - The Intermediate Value Theorem
   - Fermat's Theorem

2. (6 points) From which of the three theorems above can it be argued that if you travelled 250 miles in 3 hours on the interstate, then at some point during your trip, you broke the 75 mph speed limit? Explain.

   From which of the three theorems above can it be argued that if a rocket shot from the ground rises 10 miles in the first 5 minutes, then at some point during that time it had a vertical speed above 100 miles per hour? Explain.

   From which of the three theorems above can it be argued that if a drain pipe fills a 10 gallon bucket in 3 minutes, then at some point during that period the drain was flowing at rate of over 150 gallons per hour.

3. (15 points)

(a) If Newton's method, with initial value $x_1 = 4$, is used to approximate $\sqrt{8}$, the positive zero of $f(x) = x^2 - 8$, what are x_2 and x_3?

(b) Sketch the graph of $f(x) = x^2 - 8$. Show on your graph how Newton's Method constructs x_2 from x_1.

4. (10 points) Show that the equation $x^3 + 5x - 3 = 0$ $4x^5 + 5x^3 = 2$ $3 - 5x^3 - 6x^5 = 0$ has exactly one solution. State any theorems you use.

5. (15 points) Find the following limits. Justify your conclusions.

(a) $\lim\limits_{x\to\infty} \dfrac{x^{3/2} - 2x^3 + 1}{3\sqrt{x} - 5x^3}$ $\lim\limits_{x\to\infty} \dfrac{x^{2/3} - 2x^3 + 1}{3x^2 - 4x^3}$ $\lim\limits_{x\to\infty} \dfrac{x^{3/2} - 2x^2 + 1}{3x^4 - 5x^3}$

(b) $\lim\limits_{x\to\infty} (3x+1)\sin(\dfrac{1}{x})$ $\lim\limits_{x\to\infty} (2-5x)\sin(\dfrac{1}{x})$ $\lim\limits_{x\to\infty} (3x+1)\tan(\dfrac{1}{x})$

(c) $\lim\limits_{x\to\infty} (x - \sqrt{x^2+5x+1})$ $\lim\limits_{x\to\infty} (x - \sqrt{x^2+3x+4})$ $\lim\limits_{x\to\infty} (x - \sqrt{x^2-5x+2})$

6. (10 points) Let $f(x) = 9x + \frac{18}{x} + 2$. Let $f(x) = 5x + \frac{10}{x} + 3$. Let $f(x) = 11x + \frac{22}{x} - 10$.

   (a) Name the theorem that guarantees that f has both an absolute maximum and an absolute minimum on the interval $[-4, -1]$.

   (b) Find the absolute maximum and absolute minimum of f on $[-4, -1]$.

7. (15 points) An open (right circular cylindrical) can is to be made to contain 1000 cm^3. The tin for the sides weighs .5 grams per square centimeter, and the tin for the bottom weighs 2.0 grams per square centimeter. What should the radius of the can be to minimize its weight, and what is the minimum weight?

   A 100 foot long roll of rabbit fence is to be cut into two lengths, one to enclose a circular garden plot and the other a square garden plot. Find the maximum area that can be enclosed by the two plots.

   Use calculus to find the point on the curve $y = \sqrt{x}$ that is closest to the point $(0, 108)$.

8. (20 points) Let $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$. Let $f(x) = \dfrac{x^2}{x^2+3}$. Let $f(x) = \dfrac{x}{x^3-1}$.

(a) Find any vertical and horizontal asymptotes of the graph of f.

(b) Find the intervals of increase and decrease of f and all points $(a, f(x))$ for which $f(a)$ is a local maximum or a local minimum.

(c) Find the intervals on which f is concave upward and those on which it is concave downward. Find all inflection points $(b, f(b))$ of f.

(d) Sketch a good graph of f that plots all intercepts, local extrema, inflection points and vertical and horizontal asymptotes, and is consistent with all your answers above.