MATH 229, Exam II, Spring 2011 **NOTE:** In Spring 2011, I coordinated Math 229, and wrote the exams for all sections. The exams for different sections at different times varied slightly to prevent wholesale cheating. What you find below contains three variants of each problem used for different sections. The problems are probably a bit easier than my usual, because they were meant not only for my own students, but for the students of other lecturers.

1. (40 points) Compute the following derivatives. Simplify your answers.

   (a) $y = \dfrac{3}{10(x^4 - 3x^{+}1)^5}$

   $y = \dfrac{4}{\sqrt[3]{x^4 - 3x + 1}}$

   $y = \dfrac{5}{11(x^4 - 3x + 1)^{1/5}}$

   (b) $y = \sec^3(\frac{1}{x^2})$

   $y = \tan^4(\sqrt{x})$

   $y = \cos^4(\frac{1}{x})$

   (c) $y = (x^5 - 6x^2 + 3)\csc(2x + 5)$

   $y = (x^4 - 6x + 1)\cot(3x - 2)$

   $y = (3x^4 - 7x + 1)\cot(5 - 4x)$

   (d) $y = \sin(5x)\cos(3x)$

   $y = \cos(2x)\sin(9x)$

   $y = \sin(5x)\cos(4x)$

   (e) $y = \dfrac{5 + \frac{3}{x^2}}{7 + \frac{4}{x^2}}$

   $y = \dfrac{\frac{3}{x^2} - 6}{\frac{9}{x^2} + 5}$

   $y = \dfrac{\frac{3}{x^2} + 4}{\frac{9}{x^2} - 7}$

2. (30 points) Find all x-coordinates of points at which the given function has a horizontal tangent.

   (a) $y = \dfrac{3}{x^2} - \dfrac{6}{\sqrt{x}} + 7$

   $y = \dfrac{4}{\sqrt{x}} - \dfrac{9}{x^2} - 5$

   $y = \dfrac{10}{\sqrt{x}} - \dfrac{2}{x^3} + 6$

   (b) $y = (3x + 4)^5\sqrt{2x + 5}$

   $y = (3x + 4)^8\sqrt{2x - 9}$

   $y = (3x - 8)^{10}\sqrt{2x + 5}$

   (c) $y = \dfrac{x^4}{(9x + 8)^3}$

   $y = \dfrac{x^3}{(4x + 1)^5}$

   $y = \dfrac{x^5}{(2x + 3)^4}$

3. (10 points) Given the curve $x\sin(y) + x^2y^3 + y^2 = \frac{\pi^2}{4}$, find the following:

   (a) the formula for $\frac{dy}{dx}$

   (b) the equation of the line tangent to the curve at the point $(0, \frac{\pi}{4})$.

4. (10 points) Let $f(x) = \frac{1}{x^2}$.

   (10 points) Let $f(x) = \frac{1}{\sqrt{x}}$.

   (10 points) Let $f(x) = \frac{1}{x^3}$.

   (a) Find the formula for $L(x)$, the linearization of $f(x)$ at $a = 10$.

   Find the formula for $L(x)$, the linearization of $f(x)$ at $a = 100$.

   Find the formula for $L(x)$, the linearization of $f(x)$ at $a = 10$.

   (b) Use $L(x)$ to approximate $\frac{1}{(10.03)^2}$.

   Use $L(x)$ to approximate $\frac{1}{\sqrt{100.03}}$.

   Use $L(x)$ to approximate $\frac{1}{(10.03)^2}$.

5. (15 points) Ship A is sailing due south at 35 miles per hour, and ship B is sailing due north at 25 miles per hour. At noon, ship A is 100 miles due west of ship B. How fast is the distance between the ships changing at 4:00 PM?

   (15 points) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?