MATH 229, Bailey, Exam II NAME Solution

1. (40 points) Compute the following derivatives. Simplify your answers.

(a) $y = \frac{5}{11(x^4 - 3x + 1)^{1/5}}$ Rewrite as $y = \frac{5}{11}(x^4 - 3x + 1)^{-1/5}$

$$y' = \frac{-1}{11}(x^4 - 3x + 1)^{-6/5} \cdot (4x^3 - 3)$$

(b) $y = \sec^3(\frac{1}{x^2})$ Rewrite as $y = (\sec(x^{-2}))^3$

$$y' = 3(\sec(x^{-2}))^2 \cdot (\sec(x^{-2}) \cdot \tan(x^{-2})) \cdot -2x^{-3}$$

(c) $y = (3x^4 - 7x + 1)\cot(5 - 4x)$ Product Rule

$$y' = (3x^4 - 7x + 1) \cdot -\csc^2(5 - 4x) \cdot -4 + (12x^3 - 7)\cot(5 - 4x)$$

$$= 4(3x^4 - 7x + 1)\csc^2(5 - 4x) + (12x^3 - 7)\cot(5 - 4x)$$

(d) $y = \sin(5x)\cos(3x)$

$$y' = \sin(5x) \cdot -\sin(3x) \cdot 3 + \overset{\cos}{\cancel{\sin}}(5x) \cdot 5 \cdot \cos(3x)$$

$$= -3\sin(5x)\sin(3x) + 5\cos(5x)\cos(3x)$$

(e) $y = \frac{\frac{3}{x^2} + 4}{\frac{9}{x^2} - 7}$

$$y' = \frac{\left[\left(\frac{9}{x^2}\right) - 7\right] \cdot \left(\frac{-6}{x^3} + 0\right) - \left(\frac{-18}{x^3} + 0\right)\left(\frac{3}{x^2} + 4\right)}{\left(\frac{9}{x^2} - 7\right)^2}$$

2. (30 points) Find all x-coordinates of points at which the given function has a horizontal tangent.

(a) $y = \frac{10}{\sqrt{x}} - \frac{2}{x^3} + 6$ $= 10x^{-1/2} - 2x^{-3} + 6$

(1) $y' = -5x^{-3/2} + 6x^{-4}$

(2) $-5x^{-3/2} + 6x^{-4} = 0$ Multiply by x^4

$-5x^{5/2} + 6 = 0$

$-5x^{5/2} = -6$

$x^{5/2} = \frac{6}{5} \rightarrow x = (6/5)^{2/5}$ or $\sqrt[5]{36/25}$

(b) $y = (3x-8)^{10}\sqrt{2x+5}$

$y' = (3x-8)^{10} \cdot \frac{1}{2}(2x+5)^{-1/2} \cdot 2 + 10(3x-8)^9 \cdot 3 \cdot (2x+5)^{1/2}$

$= (3x-8)^{10}(2x+5)^{-1/2} + 30(3x-8)^9 \cdot (2x+5)^{1/2}$

$= (3x-8)^9(2x+5)^{-1/2}\left((3x-8) + 30(2x+5)\right)$

$= (3x-8)^9(2x+5)^{-1/2}(63x + 142)$

Note: $(2x+5)^{-1/2} \neq 0$

$y' = 0$ when $(3x-8)^9 = 0$, $63x + 142 = 0$

$\Rightarrow x = 8/3$ $x = -\frac{142}{63}$

(c) $y = \frac{x^5}{(2x+3)^4}$

(1) $y' = \frac{(2x+3)^4 \cdot 5x^4 - x^5 \cdot 4(2x+3)^3 \cdot 2}{(2x+3)^8}$

$= \frac{x^4(2x+3)^3(5(2x+3) - 8x)}{(2x+3)^8}$

$= \frac{x^4(10x + 15 - 8x)}{(2x+5)^3}$

$= \frac{x^4(2x+15)}{(2x+5)^3}$

(2) $y' = 0$ when

$x^4 = 0 \rightarrow x = 0$

&

$2x + 15 = 0 \rightarrow x = \frac{-15}{2}$

3. (10 points) Given the curve $x\sin(y) + x^2y^5 + y^2 = \frac{\pi^2}{4}$, find the following:

Use Implicit Differentiation

(a) the formula for $\frac{dy}{dx}$

$$x\cos(y)\cdot\frac{dy}{dx} + 1\cdot\sin(y) + x^2\cdot 5y^4\cdot\frac{dy}{dx} + 2xy^5 + 2y\cdot\frac{dy}{dx} = 0$$

Note: you can use y' instead of $\frac{dy}{dx}$

$$x\cos(y)\frac{dy}{dx} + 5x^2y^4\cdot\frac{dy}{dx} + 2y\cdot\frac{dy}{dx} = -\sin(y) - 2xy^5$$

$$\frac{dy}{dx}\left(x\cos(y) + 5x^2y^4 + 2y\right) = -\sin(y) - 2xy^5$$

$$\frac{dy}{dx} = \frac{-\sin(y) - 2xy^5}{x\cos(y) + 5x^2y^4 + 2y}$$

(b) the equation of the line tangent to the curve at the point $(0, \frac{\pi}{4})$.

Need:

(1) Point

(2) slope

(3) Use $y - y_1 = m(x - x_1)$

slope = m = y' when $x=0, y=\pi/4$ or $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-\sin(\pi/4) - 2(0)(\pi/4)^5}{0\cdot\cos(\pi/4) + 5(0)^2(\pi/4)^4 + 2(\pi/4)}$$

$$= \frac{-\sqrt{2}/2}{\pi/2} = -\frac{\sqrt{2}}{\pi}$$

So $y - \pi/4 = -\frac{\sqrt{2}}{\pi}(x - 0) \rightarrow y = \frac{-\sqrt{2}}{\pi}x + \frac{\pi}{4}$

4. (10 points) Let $f(x) = \frac{1}{x^3}$.

(a) Find the formula for $L(x)$, the linearization of $f(x)$ at $a = 10$.

Note: $L(x) = f(a) + f'(a)(x - a)$

(1) $a = 10$

(2) $f(a) = f(10) = \frac{1}{10^3}$

(3) $f'(x) = \frac{-3}{x^4}$

(4) $f'(10) = \frac{-3}{10^4}$

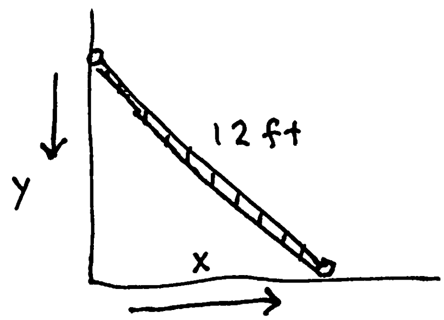
So $L(x) = \frac{1}{10^3} + \frac{-3}{10^4}(x - 10)$

(b) Use $L(x)$ to approximate $\frac{1}{(10.03)^3}$.

$$\frac{1}{10.03^3} = f(10.03) \approx L(10.03) = \frac{1}{1000} + \frac{-3}{10000}(10.03 - 10)$$

$$\approx .000991$$

5. (15 points) A 12 foot ladder is sliding down a wall. At the time when the foot of the ladder is 3 feet from the wall, the top of the ladder is sliding down the wall at a rate of 2 f/s. How fast is the foot of the ladder sliding across the floor at this time?



Know: $\frac{dy}{dt} = -2$ ft/s

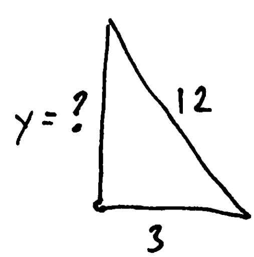
want $\frac{dx}{dt}$ when $x = 3$ ft.

(1) Formula relating x and y : $x^2 + y^2 = 12^2$ Not 2·12

(2) Differentiate : $\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y \cdot \frac{dy}{dt} = 0$

$$x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} = 0$$

(3) we know $\frac{dy}{dt} = -2$, $x = 3$, but I don't know y. But at this moment, we have the following triangle



$$y^2 + 3^2 = 12^2$$
$$y^2 = 144 - 9$$
$$y^2 = 135$$
$$y = \sqrt{135}$$

(4) Go back to $x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} = 0$

$\rightarrow 3\left(\frac{dx}{dt}\right) + \sqrt{135} \cdot -2 = 0$

$\rightarrow \frac{dx}{dt} = \frac{2\sqrt{135}}{3} \approx 7.746$ ft/s