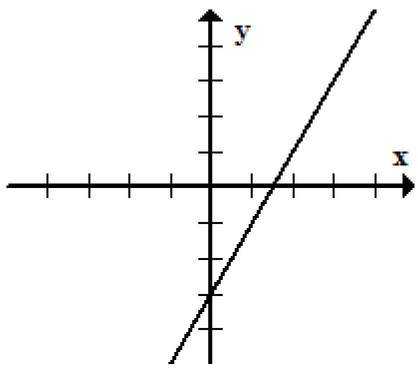
1.2 Types of Functions

Let's run through all the different types of functions you'll see. You should have most of these memorized. You should also know the basic properties of each of the functions.

1. Linear: $f(x) = mx + b$

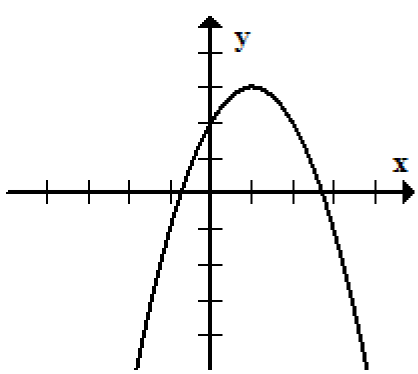


Graph 1.2

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{f(b) - f(a)}{b - a}$

Get used to the second version.

2. Quadratic: $f(x) = ax^2 + bx + c$

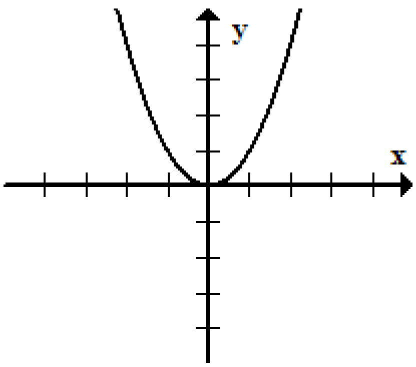


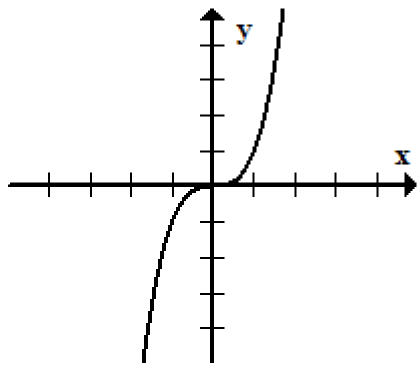
Graph 1.3

Warning: Know how to factor quadratics. It comes up.

3. General Polynomial: $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + ... + a_1 x + a_0$

4. Power Functions: $f(x) = x^n$



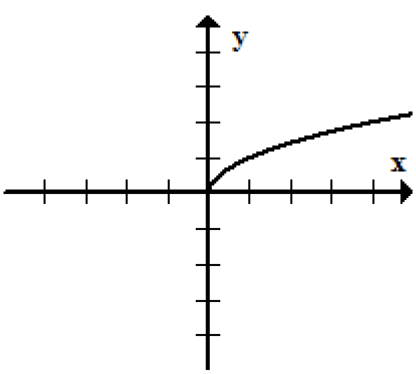


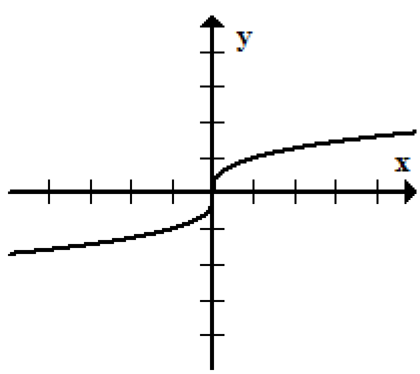
Even Power

Odd Power

5. Root Functions: $f(x) = \sqrt{x},\ \sqrt[3]{x},\ \sqrt[4]{x},\ ...,\ \sqrt[n]{x}$

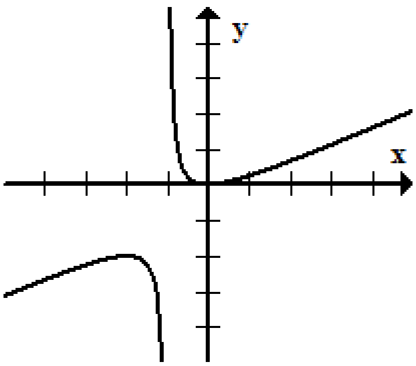
Or their equivalent fractional exponents: $x^{1/2}$, $x^{1/3}$, $x^{1/4}$, ..., $x^{1/n}$



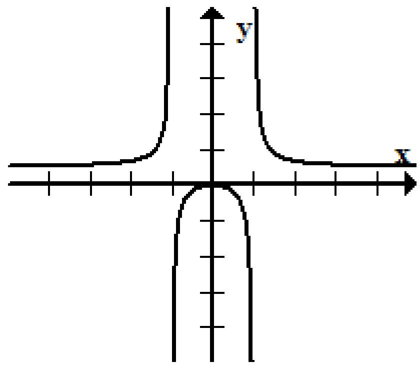


6. Rational Functions: $f(x) = \dfrac{P(x)}{Q(x)}$, where P and Q are polynomials.

   Note: These do have domain issues when $Q(x) = 0$. Watch out for those. Rational functions tend to look like this.



Graph 1-6a



Graph 1-6b

Rational Functions tend to have horizontal and vertical asymptotes. Our *calculus* way of defining these will be many pages later. From algebra you had shortcuts to determine these.

1.2.1 Horizontal Asymptotes

(a) If the degree of $P(x)$ (top polynomial) is bigger than the degree of $Q(x)$ (bottom polynomial), then there is no horizontal asymptote. These would be oblique asymptotes, which sadly are skipped over in algebra classes.

   Graph 1-6a: $f(x) = \dfrac{x^2}{2x+2}$ is example of a rational function with an oblique asymptote. Notice how the graph will go to $\pm\infty$ when x gets really large.

(b) If the degree of $P(x)$ (top polynomial) is the same as the degree of $Q(x)$ (bottom polynomial), then the horizontal asymptote is the ratio of the leading coefficients.

Example 1.1. $f(x) = \dfrac{4x^3 - 15x + 7}{1 - 3x^3}$ The horizontal asymptote will be $y = \dfrac{4}{3}$

(c) If the degree of $P(x)$ is smaller than the degree of $Q(x)$, then the horizontal asymptote is $y = 0$. This one is easy!

1.2.2 Vertical Asymptotes

(a) Make sure $f(x) = \dfrac{P(x)}{Q(x)}$ is simplified.

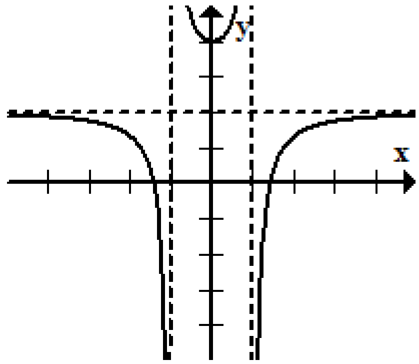
(b) Find what x-values make $Q(x) = 0$. These are potential vertical asymptotes. Suppose $Q(a) = 0$.

(c) If those x-values make $P(x) \neq 0$, then $x = a$ is a vertical asymptote.

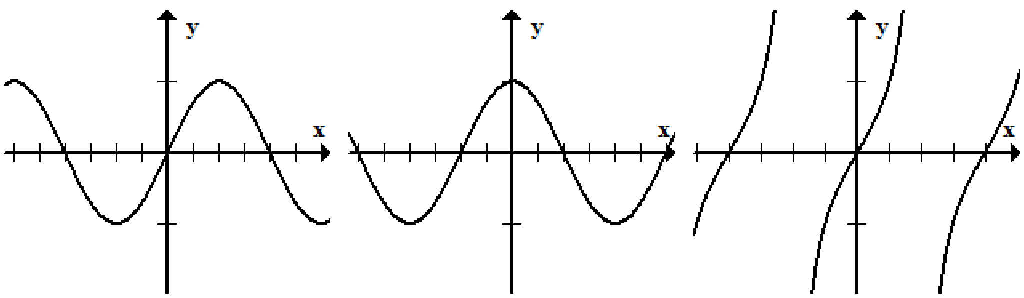
Example 1.2. $f(x) = \dfrac{x^2 - 4}{x^2 - 1}$

Since $x^2 - 1 = 0$ when $x = 1, -1$, $x = 1$ and $x = -1$ are potential vertical asymptote. Since $x^2 - 4 \neq 0$ when $x = 1$ or $x = -1$, we know $x = 1$ and $x = -1$ are vertical asymptotes.

Here's the graph



7. Trig Functions



Graph 1-8 $f(x) = \sin(x)$ $f(x) = \cos(x)$ $f(x)\tan(x)$