1.4 Transformations

When we talk about transformations in this class we are referring to shifting, stretching, compressing, and rotating *basic* functions. What is a *basic* function? Here's a list.

1. $y = x^2$ or $y = x^3$
2. $y = \sqrt{x}$ or $y = \sqrt[3]{x}$
3. $y = \dfrac{1}{x}$
4. $y = |x|$
5. $y = \cos(x)$ or $y = \sin(x)$

Let's go through the list of our transformations.

For $c > 0$. Let $f(x)$ be a *basic* function.

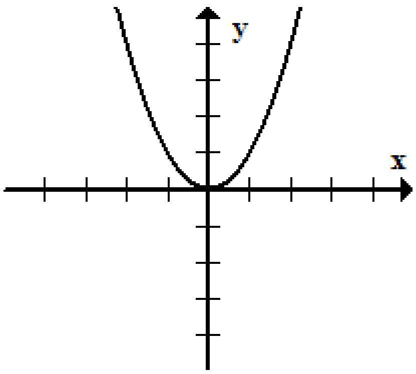
Function	Effect	Example
$y = f(x) + c$	Vertical Shift up c units	$y = x^2 + 1$
$y = f(x) - c$	Vertical Shift down c units	$y = \sqrt{x} - 4$
$y = f(x - c)$	Horizontal Shift right c units	$y = \|x - 4\|$
$y = f(x + c)$	Horizontal Shift left c units	$y = \dfrac{1}{x+3}$

Now...on to some examples

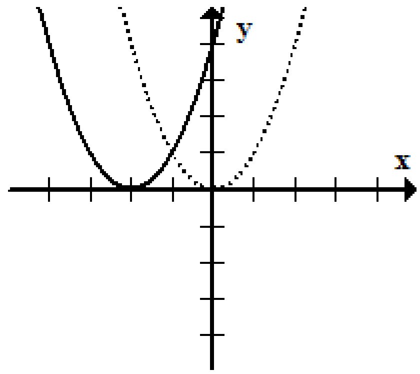
Example 1.5.

1. Sketch $y = (x+2)^2$

   This is a horizontal shift *left* 2 units.



$y = x^2$



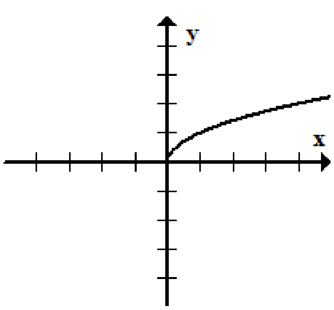
$y = (x+2)^2$

2. Sketch $y = \sqrt{x-1} + 2$

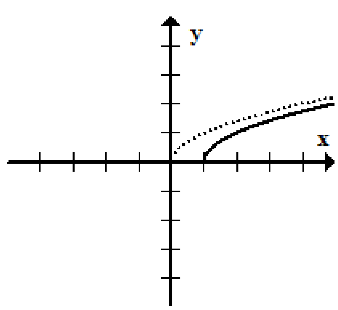
   This has two transformations. Start with the one closest to the x. So it goes...

   (a) Shift *right* 1 unit

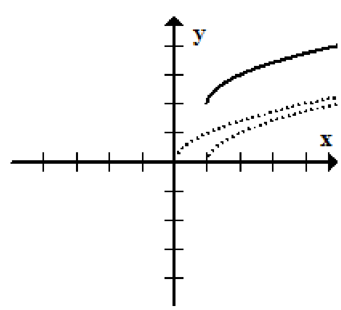
   (b) Shift *up* 2 units



$y = \sqrt{x}$



$y = \sqrt{x-1}$



$y = \sqrt{x-1} + 2$

So those were the *shifts*. Let's move on to the *stretches* and *compressions*

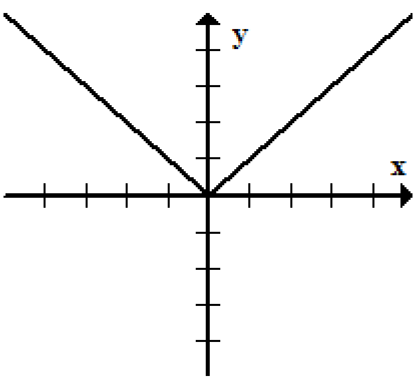
For $c > 1$. Let $f(x)$ be a basic function.

Function	Effect	Example
$y = cf(x)$	Stretch vertically by a factor of c	$y = 3\|x\|$
$y = \frac{1}{c}f(x)$	Compress vertically by a factor of c	$y = \frac{1}{4}x^2$
$y = f(cx)$	Compress Horizontally	$y = \cos(4x)$
$y = f\left(\frac{1}{c}x\right)$	Stretch horizontally	$y = \sin\left(\frac{x}{2}\right)$

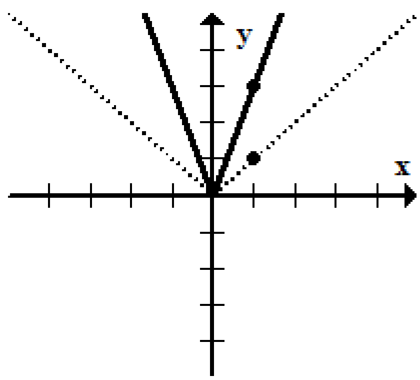
Example 1.6.

1. Sketch $y = 3|x|$

   This is a vertical stretch by a factor of 3.

   $y = |x|$

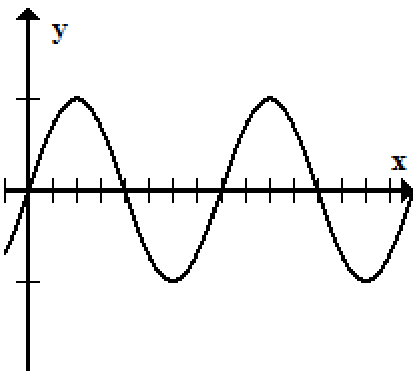
   

   $y = 3|x|$

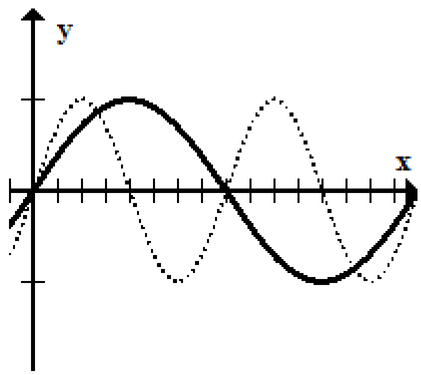
   The way this works is take any point on the graph and multiply the y-value by a factor of 3. Note how the point (1,1) is now at (1,3).

2. Sketch $y = \sin\left(\frac{x}{2}\right)$

I think it's easiest to see why this is a horizontal stretch when you use sin(x). Recall that sin(x) has a period of 2π. It means it will complete a full period between 0 to 2π. The left graph shows 2 full periods. When you multiple the 'inside' by 1/2, it means you'll complete 1/2 as many periods, i.e., the period length should be "larger" or "stretched." So in this case, the new graph should have only 1 period (half as many as the first graph). Let's take a look.



$y = \sin(x)$



$y = \sin\left(\frac{x}{2}\right)$

Try to sketch:

1. $y = \frac{1}{3}\sqrt{x}$

2. $y = (2x)^2$

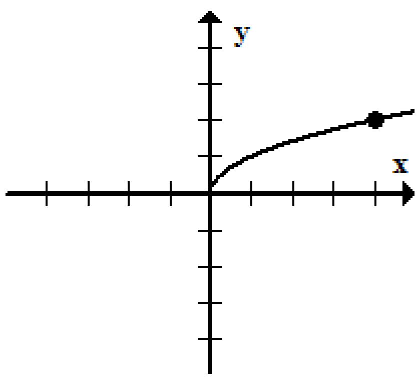
Our last transformations are reflections.

Function	Effect	Example
$y = -f(x)$	Flip over y-axis	$y = -x^2$
$y = f(-x)$	Flip over x-axis	$y = \sqrt{-x}$

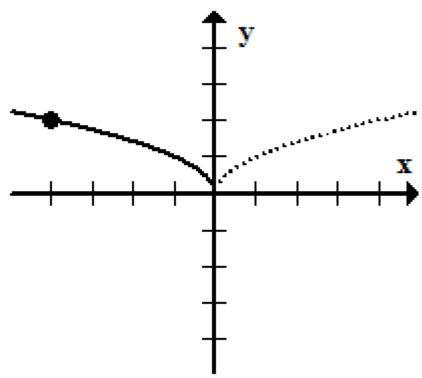
Example 1.7.

1. Sketch $y = \sqrt{-x}$

   The basic function here is $y = \sqrt{x}$. The (-) sign inside of the basic function, so it's a reflection over the x-axis. Another way to look at it is follow some points on the original function and the transformed function.



$y = \sqrt{x}$



$y = \sqrt{-x}$

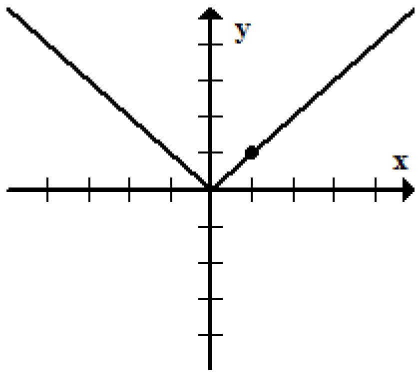
Example 1.8. For the final sketch, let's add a bunch of transformations together. Sketch

$$y = -2|x - 1| + 3$$

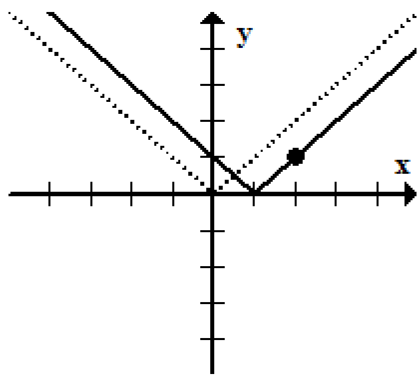
To complete this graph, let's map out the steps. Always start with the transformations closest to x.

1. Shift 1 unit to the right
2. Vertical Stretch by a factor of 2
3. Reflect over y-axis
4. Shift up 3 units

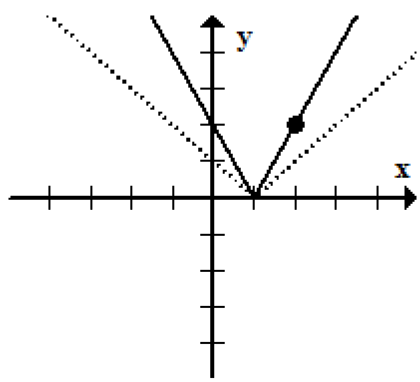
Let's get started. I will plot the point (1,1). We will follow the point as it gets transformed.



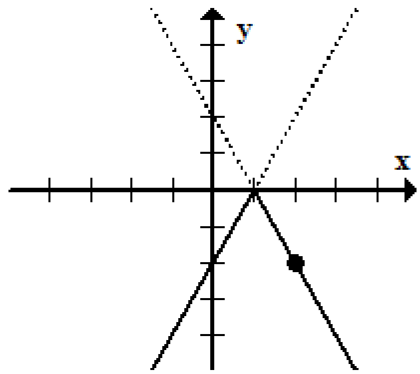
1. Shift 1 unit to the right.



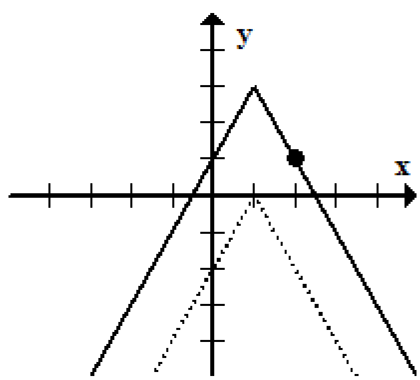
2. Vertical Stretch by a factor of 2.



3. Reflect over the y-axis.



4. Shift up 3 units



Alright! We are now done with transformations.