MATH 229 FINAL EXAM Name ______________________

FALL 2014 Score ______________________

Please show all your work and display your answers clearly.

1. (18 pts.) Find the following limits. Justify your answers.

(a) $\lim_{x\to 1} \frac{\sqrt{x+1}-\sqrt{2}}{x-1}$

(b) $\lim_{x\to -\infty} \frac{2+x-3x^3}{4+5x^3}$.

(c) $\lim_{x\to 0} \frac{x\cos(2x)}{\sin(-3x)}$

2. (24 pts.) Compute the derivatives of the following functions.

(a) $y = \sqrt[3]{x^2} - \csc(2x) + \frac{1}{\pi}$

(b) $y = \left(\frac{x^2+1}{\sin x}\right)^3$

(c) $y = x^3 \tan(1-x)$

(d) $y = \int_x^{\pi/2} \frac{\sin s}{1+s^2}\, ds$

3. (24 pts.) Find the following indefinite integrals and definite integrals.

(a) $\int \sqrt[3]{x}(x+4)\, dx$

(b) $\int \frac{x^2}{\sqrt{x^3-5}}\, dx$

(c) $\displaystyle\int_0^{\pi/4} \tan x \sec^2 x \, dx$

(d) $\displaystyle\int_0^{3} |x-2| \, dx$

4. (12 pts.) (a) State the limit definition of the derivative $f'(x)$ of a function $f(x)$.

(b) Use the definition in (a) to compute $f'(2)$ for the function $f(x) = \dfrac{1}{x}$. (No credit will be given for any other method.)

5. (10 pts.) Let $f(x)$ be defined by $f(x) = \begin{cases} 3x^2 - 2x, & x \geq 1 \\ -4 + x, & x < 1 \end{cases}$. Is $f(x)$ continuous at $x = 1$? Is $f(x)$ differentiable at $x = 1$? Justify your answers.

6. (10 pts.) Assume that the equation $x^2 + xy + y^2 = 1$ defines a function $y = f(x)$. Find $\frac{dy}{dx}$.

7. (10 pts.) Find an equation for the tangent line to the curve $y = x^4 - 7x + 2$ at the point where $x = 1$.

8. (18 pts.) The following information is given for a function $f(x)$:

(i) f is defined on the whole real number line except at $x = 1$;

(ii) $f'(x) = -\dfrac{x+1}{(x-1)^3}$, $\quad f''(x) = \dfrac{2x+4}{(x-1)^4}$, $\quad x \neq 1$;

(iii) $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to 1} f(x) = \infty$;

(iv) $f(0) = 0$, $\quad f(-1) = -1/4$, $\quad f(-2) = -2/9$, $\quad f(2) = 2$.

Use the above information to fill in the following blanks:

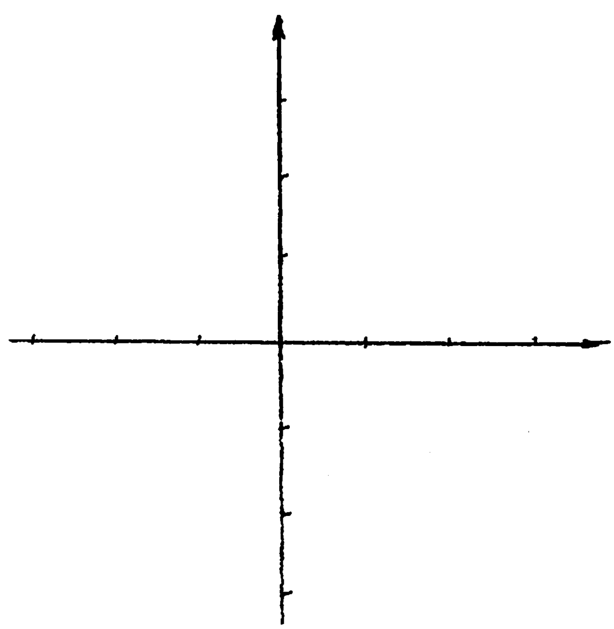
(a) intervals where f is increasing ____________________
intervals where f is decreasing ____________________

(b) intervals where f is concave up ____________________
intervals where f is concave down ____________________

(c) local extreme points of f ____________________
points of inflection of f ____________________

(d) horizontal asymptote of f ____________________
vertical asymptote of f ____________________.

Then sketch the graph of the function f based on the results in (a)-(d) and using the points given in (iv).



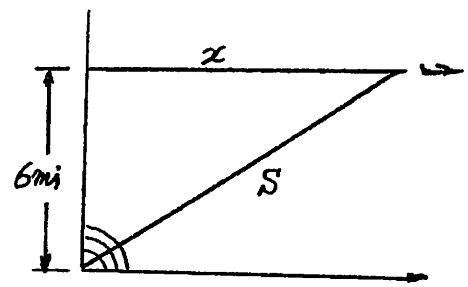
9. (10 pts.) A curve is defined by the function $f(x) = \dfrac{x - 3x^2}{x^2 + x - 6}$.

(a) Find the horizontal asymptote of this curve.

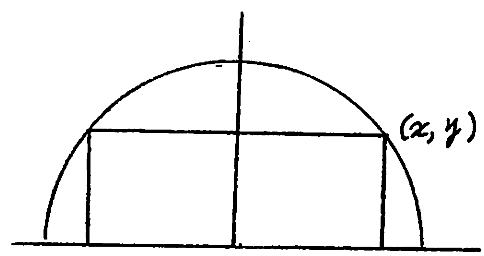
(b) Find the vertical asymptotes of this curve.

10. (10 pts.) Find the maximum and minimum values of the function $y = \dfrac{x}{x^2 + 1}$ on the interval $[0, 3]$.

11. (12 pts.) An airplane is flying at an altitude of 6 miles and passes directly over a radar antenna (see figure). When the plane is 10 miles away ($s = 10$), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the plane?



12. (12 pts.) A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?



13. (10 pts.) A particle moves along a straight line with the acceleration $a(t) = \sin t - 3$, the initial position $s(0) = 1$, and the initial velocity $v(0) = -1$. Find the position function $s(t)$.

14. (10 pts.) Approximate the integral $\int_1^2 \frac{1}{x}\,dx$ by using a Riemann sum with a partition of three subintervals of equal length and with left endpoints as the sample points. Is this a lower or upper estimate?

15. (10 pts.) Use the limit definition of definite integral in the form of $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$, where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$, to evaluate the definite integral $\int_1^3 (4-x)\,dx$.

(Formulas: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$)