

Name \_\_\_\_\_

Math 229  
Final Exam  
Fall 2013

1. (10 pts) Let  $f(x) = \sqrt{5x+2}$ . Find  $f'(x)$  by the limit definition of the derivative.

2. (10 pts) Let  $f(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ C & x = 2 \\ 4x & x > 2 \end{cases}$

(a) Does  $\lim_{x \rightarrow 2} f(x)$  exist? If yes, what is it? If not, explain.

(b) Is there a value of  $C$  which makes  $f(x)$  continuous at  $x = 2$ ? If yes, what is it? If not, explain.

3. (21 pts) Evaluate the following limits. Show your work!

(a)  $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x^2 - 16}$

(b)  $\lim_{t \rightarrow 5} \frac{\frac{1}{t} - \frac{1}{5}}{t - 5}$

(c)  $\lim_{\theta \rightarrow 0} \frac{\sin(3\theta)}{\theta + \sin(\theta)}$  Hint: multiply and divide by  $\frac{1}{\theta}$ .

4. (28 pts) Find the first derivative of the following functions. Do not simplify your answer.

(a)  $\frac{x^2 - 4x + 3}{\sqrt{x}}$

(b)  $\sin^3(4\theta)$

(c)  $t^2 \tan^2(4t)$

(d)  $\sqrt{x^2 + \cos x}$

5. (10 pts) Find the tangent line to the graph of  $x^3 + y^3 + 3xy = 15$  at the point  $(1, 2)$ . Put your answer in slope-intercept form.

6. (10 pts) What are the maximum and minimum values of  $f(x) = x + 2 \cos x$  on the interval  $[0, \pi]$ ?

7. (30 pts) Let  $f(x) = \frac{x}{x^2-4}$ . Then  $f'(x) = \frac{-(x^2+4)}{(x^2-4)^2}$  and  $f''(x) = \frac{2x(x^2+12)}{(x^2-4)^3}$

(a) What are the horizontal asymptotes of  $f(x)$ ?

(b) What are the vertical asymptotes of  $f(x)$ ?

(c) Where is  $f(x)$  increasing? Decreasing?

(d) Where is  $f(x)$  concave up? Concave down?

(e) Sketch a graph of  $y = f(x)$ . Label extrema and points of inflection.

8. (15 pts) A rectangle with side lengths  $x$  and  $y$  has an area of 125. What is the maximum possible value of  $x^2 + 2y$  for such a rectangle?

9. (15 pts) A metallic cylinder is placed in a machine press. During the press, the volume of the cylinder stays constant, but the height is decreasing at the rate of 3 inches per hour. How fast is the radius changing when the radius is 20 inches and the height is 30 inches?

10. (10 pts) Use Riemann sums and right hand endpoints to approximate  $\int_1^9 \frac{1}{1+x^2} dx$ . Use  $n = 4$  rectangles.

11. (13 pts) Let  $f(x) = \int_0^x \sqrt[3]{1+t^2} dt$

(a) What is  $f'(x)$ ? Explain.

(b) Use this to find the derivative of  $g(x) = \int_0^{\tan x} \sqrt[3]{1+t^2} dt$ .

12. (28 pts) Evaluate the following integrals:

(a)  $\int \frac{x^3 - 6x^2 + x + 2}{\sqrt{x}} dx$

(b)  $\int_0^4 \frac{x dx}{\sqrt{x^2 + 9}}$

(c)  $\int_0^{\frac{\pi}{2}} \cos x dx$

(d)  $\int \cos^3 x \sin x dx$