Name:

Math 229
Mock Final Exam Solution

Disclaimer: This mock exam is for practice purposes only. No graphing calulators $\geq$ TI-89 is allowed on this test. Be sure that all of your work is shown and that it is well organized and legible.

This mock exam does not cover all the material that can be tested on the final exam. Please review all exams, quizzes, and homeworks.

Good luck!

1. (10pts) True or False. Cirlce your answer.

   (a) **T** f $f(x) = |x|$ is continuous at $x = 0$

   (b) t **F** $f(x) = |x|$ is differentiable at $x = 0$

   (c) t **F** $f(x) = \sqrt{x}$ is differentiable at $x = 0$

   (d) t **F** If a is a critical value of f, then f must have a maximum or minimum at $x = a$.

   (e) t **F** If $f''(a) = 0$, then a must be an inflection point.

2. (24 points) Find the following limits. If the limit is infinite, write ∞ or $-\infty$.

   (a) $\lim_{x \to 2} \frac{x^2 - 2x}{x^2 + 4x - 12}$

$$\lim_{x \to 2} \frac{x^2 - 2x}{x^2 + 4x - 12} = \lim_{x \to 2} \frac{x(x-2)}{(x-2)(x+6)} = \lim_{x \to 2} \frac{x}{x+6} = \frac{2}{2+6} = \frac{2}{8} = \frac{1}{4}$$

   (b) $\lim_{x \to 0} \frac{\sin(x)}{5x + 8x^2}$

$$\lim_{x \to 0} \frac{\sin(x)}{x(5+8x)} = \lim_{x \to 0} \frac{\sin(x)}{x} \cdot \frac{1}{5+8x} = 1 \cdot \frac{1}{5+8(0)} = \frac{1}{5}$$

Note:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

   (c) $\lim_{x \to 6^-} \frac{x+6}{x(x-6)}$

If you plug in $x = 6$, you get $\frac{12}{6(0)} = \frac{12}{0}$. This is bad. Since it's a one-sided limit, we can set up a table to find out what's happening.

Let $f(x) = \frac{x+6}{x(x-6)}$. Let's plug numbers in that are approaching $x = 6$ from the left.

$x = 5.5$	$f(5.5) = -4.18$
$x = 5.9$	$f(5.9) = -20.17$
$x = 5.99$	$f(5.99) = -200.17$
$x = 5.999$	$f(5.999) = -2000.17$
$x = 5.99999$	$f(5.99999) = -200000.17$

Based on this, as $x \to 6^-$, the y-values are approaching $-\infty$. Therefore,

$$\lim_{x \to 6^-} \frac{x+6}{x(x-6)} = -\infty$$

Note: If $x \to 6$, without a direction, you must check both the left and right sided limits.

(d) $\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2}{x-1}$

If you plug in $x = 1$, you get $\frac{0}{0}$. This means we have some work to do. Since we have a radical on top, let's multiply the top and bottom by the conjuate of the numerator.

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2}{x-1} &= \lim_{x\to 1} \frac{\sqrt{x^2+3x}-2}{x-1} \cdot \frac{\sqrt{x^2+3x}+2}{\sqrt{x^2+3x}+2} \\
&= \lim_{x\to 1} \frac{(x^2+3x)-4}{(x-1)(\sqrt{x^2+3x}+2)} \\
&= \lim_{x\to 1} \frac{x^2+3x-4}{(x-1)(\sqrt{x^2+3x}+2)} \\
&= \lim_{x\to 1} \frac{(x-1)(x+4)}{(x-1)(\sqrt{x^2+3x}+2)} \\
&= \lim_{x\to 1} \frac{x+4}{(\sqrt{x^2+3x}+2)} \\
&= \frac{1+4}{\sqrt{1^2+3(1)}+2} \\
&= \frac{5}{4}
\end{aligned}$$

3. (10 points) Let

$$f(x) = \begin{cases} -x^2, & \text{if } x < 0 \\ \cos(x), & \text{if } 0 \le x < \pi/2 \\ x - \pi/2, & \text{if } x \ge \pi/2 \end{cases}$$

(a) Find $\lim_{x\to 0} f(x)$, if it exists.

i. Check both the left and right hand limit.

ii. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x^2) = 0$

iii. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \cos(x) = \cos(0) = 1$

iv. Since the $\lim_{x\to 0^-} f(x) \ne \lim_{x\to 0^+} f(x)$, the general limit $\lim_{x\to 0} f(x)$ does not exist.

(b) Is $f(x)$ continuous at $x = 0$

There are three condtions to satisify for a function to be continuous at $x = 0$.

i. Does $f(0)$ exist? Yes, and $f(0) = \cos(0) = 1$.

ii. Does $\lim_{x\to 0} f(x)$ exist? No, we found this out in (a).

Since it fails one of the conditions to be continuous, we conclude that $f(x)$ is NOT continuous at $x = 0$.

(c) Is $f(x)$ continuous at $x = \pi/2$

There are three condtions to satisify for a function to be continuous at $x = \pi/2$.

i. Does $f(\pi/2)$ exist? Yes, and $f(\pi/2) = \pi/2 - \pi/2 = 0$.

ii. Does $\lim_{x\to\pi/2} f(x)$ exist? We need to check the left and right sided limits first.

$$\lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^-} \cos(x) = \cos(\pi/2) = 0$$
$$\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^+} x - \pi/2 = \pi/2 - \pi/2 = 0$$

Therefore, the general limit $\lim_{x\to\pi/2} f(x)$ exists and equals 0.

iii. Does $\lim_{x\to\pi/2} f(x) = f(\pi/2)$? Yes, we can see that from parts (i) and (ii).

Since $f(x)$ satisfies all three conditions, $f(x)$ is continuous at $x = \pi/2$.

4. (10 points) Let $f(x) = \dfrac{-16}{x}$. Use the limit definition to find $f'(2)$. No credit for any other method.

There are two ways of finding the derivative function using the limit definition.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

Since the problem tells us to find the slope at $x = 2$, we know $a = 2$. But since you may be asked to find only the derivative, we'll find that first and then plug in $x = 2$.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{\dfrac{-16}{x+h} - \dfrac{-16}{x}}{h} \end{aligned}$$

Multiply top and bottom by the common denominator $x(x+h)$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\frac{-16}{x+h} - \frac{-16}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)} \\
&= \lim_{h\to 0} \frac{-16x + 16(x+h)}{h \cdot x(x+h)} \\
&= \lim_{h\to 0} \frac{-16x + 16x + 16h}{h \cdot x(x+h)} \\
&= \lim_{h\to 0} \frac{16h}{h \cdot x(x+h)} \\
&= \lim_{h\to 0} \frac{16}{x(x+h)} \\
&= \frac{16}{x(x+0)} \\
&= \frac{16}{x^2}
\end{aligned}
$$

If you want to do it with the other version, do the following

$$
\begin{aligned}
f'(a) &= \lim_{x\to a} \frac{f(x) - f(a)}{x-a} \\
&= \lim_{x\to a} \frac{\frac{-16}{x} - \frac{-16}{a}}{x-a} \\
&= \lim_{x\to a} \frac{\frac{-16}{x} + \frac{16}{a}}{x-a} \cdot \frac{x \cdot a}{x \cdot a} \\
&= \lim_{x\to a} \frac{-16a + 16x}{(x-a) \cdot x \cdot a} \\
&= \lim_{x\to a} \frac{16(x-a)}{(x-a) \cdot ax} \\
&= \lim_{x\to a} \frac{16}{ax} \\
&= \frac{16}{a^2}
\end{aligned}
$$

To find $f'(2)$, plug in $x = 2$ or $a = 2$ depending on which version you used.

$$f'(2) = \frac{16}{2^2} = 4$$

5. (8 points) Find the equation of the tangent line to the graph $f(x) = \cos(x) + x$ at the point $(0, 1)$.

   (a) To find the equation of the tangent line, you need two things: a point and the slope at that point.

   (b) We already know the point. It's $(0, 1)$.

   (c) To find the slope, we find $f'(x)$

   $$f'(x) = -\sin(x) + 1$$

   (d) Find the slope at (0,1):

   $$m = f'(0) = -\sin(0) + 1 = 1$$

   (e) Use the point-slope formula: $y - y_1 = m(x - x_1)$

   $$y - 1 = 1(x - 0)$$
   $$y - 1 = 1x$$
   $$y = 1x + 1$$

6. (8 points) Let $f(x) = 3x^5 - 5x^3$. Use Newton's Method with thte initial approxiation $x_1 = 1.5$. Find x_2 and x_3.

   The formula for Newton's Method is

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

   (a) Find $f'(x)$:

   $$f'(x) = 15x^4 - 15x^2$$

   (b) Write out Newton's Method:

   $$x_{n+1} = x_n - \frac{3x_n^5 - 5x_n^3}{15x_n^4 - 15x_n^2}$$

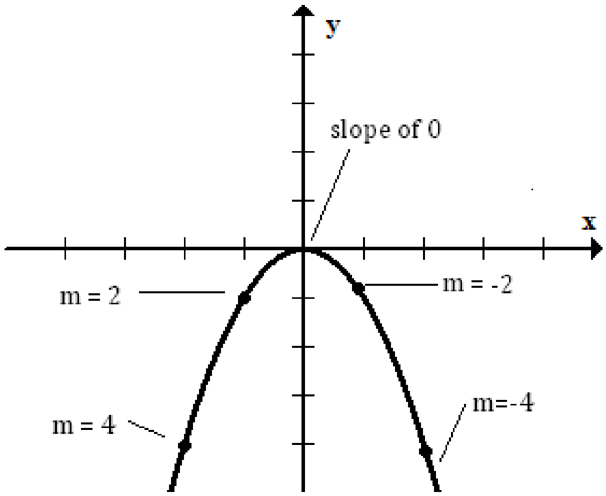
   (c) Set up a table

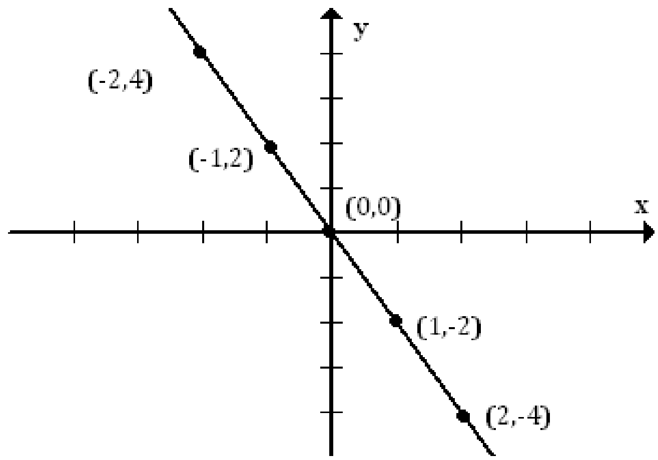
x_n	approxmiation
x_1	1.5
x_2	$1.5 - \frac{f(1.5)}{f'(1.5)} = 1.36$
x_3	$1.36 - \frac{f(1.36)}{f'(1.26)} = 1.3014$

7. (8 points) In each part, the graph of a function is given. Draw a graph of the derivative of this function in the adjacent coordinate system.

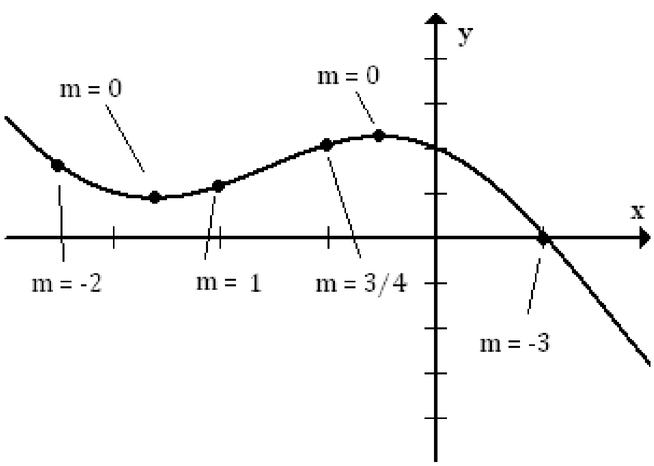
Note: The most important thing is to remember that slopes on the function y are the y-values on y'. Ex. If at $x = 3$, you have a slope of -2, then you plot the point $(3, -2)$ on the derivative's graph.

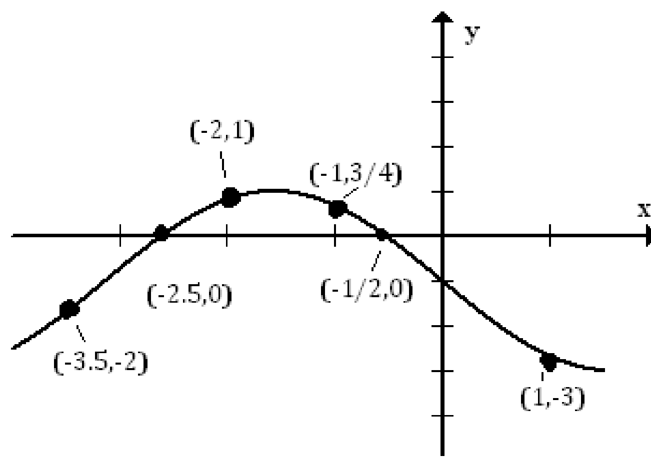
(a)





(b)





With this one you need to estimate the slopes the best you can. You may be off, but as long as the other slopes are ok in relationship to each other, you'll be fine.

8. (24 points) Find the derivative of the following functions. You do not need to simplify.

(a) $f(x) = \sqrt{\sin(x^2)}$

i. Rewrite the function as $f(x) = \left[\sin(x^2)\right]^{1/2}$

ii. This is a chain rule. I'm going to take it one step at a time. Some of you are able to go straight to the last step.

$$\begin{aligned} f'(x) &= \frac{1}{2}\left[\sin(x^2)\right]^{-1/2} \cdot \frac{d}{dx}\left[\sin(x^2)\right] \\ &= \frac{1}{2}\left[\sin(x^2)\right]^{-1/2} \cdot \cos(x^2) \cdot \frac{d}{dx}\left[x^2\right] \\ &= \frac{1}{2}\left[\sin(x^2)\right]^{-1/2} \cdot \cos(x^2) \cdot (2x) \end{aligned}$$

(b) $f(x) = (2x+7)^{10}(5x-3)^4$

This is a product rule first, with some chain rules inside.

$$\begin{aligned} f'(x) &= (2x+7)^{10} \cdot \frac{d}{dx}\left[(5x-3)^4\right] + (5x-3)^4 \cdot \frac{d}{dx}\left[(2x+7)^{10}\right] \\ &= (2x+7)^{10} \cdot \left[4(5x-3)^3 \cdot 5\right] + (5x-3)^4 \cdot \left[10(2x+7)^9 \cdot 2\right] \\ &= 20(2x+7)^{10}(5x-3)^3 + 20(2x+7)^9(5x-3)^4 \end{aligned}$$

(c) $f(x) = \dfrac{2x^6 - 7x + 5}{x^{4/3} + x^{-1/2}}$

This is a straight forward quotient rule.

$$\begin{aligned} f'(x) &= \frac{(x^{4/3} + x^{-1/2}) \cdot \frac{d}{dx}\left[2x^6 - 7x + 5\right] - (2x^6 - 7x + 5) \cdot \frac{d}{dx}\left[x^{4/3} + x^{-1/2}\right]}{(x^{4/3} + x^{-1/2})^2} \\ &= \frac{(x^{4/3} + x^{-1/2}) \cdot (12x^5 - 7) - (2x^6 - 7x + 5) \cdot \left(\frac{4}{3}x^{1/3} - \frac{1}{2}x^{-3/2}\right)}{(x^{4/3} + x^{-1/2})^2} \end{aligned}$$

(d) $f(x) = \csc^2(x) + x^9$

It might be helpful to rewrite $f(x)$ as $f(x) = (\csc(x))^2 + x^9$. It will help you identify $\csc^2(x)$ as a chain rule.

$$\begin{aligned} f'(x) &= 2\left(\csc(x)\right)^1 \cdot \frac{d}{dx}\left[\csc(x)\right] + 9x^8 \\ &= 2\left(\csc(x)\right) \cdot \left(-\csc(x)\cot(x)\right) + 9x^8 \\ &= -2\csc^2(x)\cot(x) + 9x^8 \end{aligned}$$

9. (8 points) Find $\dfrac{d^2y}{dx^2}$ if $y = x\sec(x)$

Note: $\dfrac{d^2y}{dx^2}$ is the same thing as saying, "Find the second derivative of y" or find y''.

$$\begin{aligned}
\frac{dy}{dx} &= x \cdot \frac{d}{dx}(\sec(x)) + \sec(x) \cdot \frac{d}{dx}(x) \\
&= x \cdot (\sec(x)\tan(x)) + \sec(x) \\
&= \sec(x) \cdot (x\tan(x) + 1)
\end{aligned}$$

To find $\frac{d^2y}{dx^2}$, we take the derivative again.

$$\begin{aligned}
\frac{d^2y}{dx^2} &= \sec(x) \cdot \frac{d}{dx}[x\tan(x) + 1] + (x\tan(x) + 1) \cdot \frac{d}{dx}(\sec(x)) \\
&= \sec(x) \cdot \big(x\sec^2(x) + \tan(x) + 0\big) + (x\tan(x) + 1) \cdot \sec(x)\tan(x)
\end{aligned}$$

Note: I had to differentiate $x\tan(x)$, which is another product rule.

10. (6 points) Use implicit differentiation to find $\frac{dy}{dx}$ if $y + x^3 + y^3 = 3.14159$

Note: With implicit differentiation, y is a function of x. Also, instead of $\frac{dy}{dx}$ I will use y'. My goal for this problem was get you familiar with the concept of implicit differentation again. Your exam question will probably be more difficult.

(a) Differentiate both sides with respect to x

$$y' + 3x^2 + 3y^2 \cdot y' = 0$$

(b) Solve for y'

$$\begin{aligned}
y' + 3x^2 + 3y^2 \cdot y' &= 0 \\
y' + 3y^2 \cdot y' &= -3x^2 \\
y'(1 + 3y^2) &= -3x^2 \\
y' &= \frac{-3x^2}{1 + 3y^2}
\end{aligned}$$

11. (10 points) Find the critical points of the function $f(x) = \frac{x^2 - 3}{x - 2}$. Determine whether each critical point is a local maximum, minimum, or neither.

(a) In order to find critical values, we need to find $f'(x)$.

$$
\begin{aligned}
f'(x) &= \frac{(x-2)(2x)-(x^2-3)(1)}{(x-2)^2} \\
&= \frac{2x^2-4x-x^2+3}{(x-2)^2} \\
&= \frac{x^2-4x+3}{(x-2)^2} \\
&= \frac{(x-1)(x-3)}{(x-2)^2}
\end{aligned}
$$

(b) Critical values occur when $f'(x)=0$ and when $f'(x)$ does not exist.

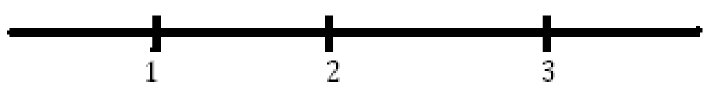
$$f'(x)=0 \text{ when } (x-1)(x-3)=0$$

$$x=1, x=3$$

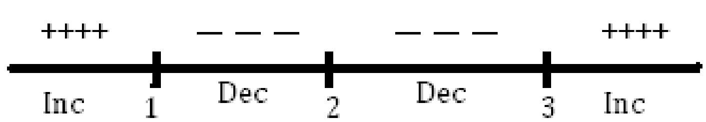
$$f'(x) \text{ does not exist when } (x-2)^2=0$$

$$x=2$$

(c) We use the following number line to determine when the function is increasing or decreasing. Plot the critical values on this number line.



(d) Choose a number from each region of the number line. Plug it into $f'(x)$ and determine if it's postive $(+)$ or negative $(-)$.



I used

$$
\begin{aligned}
f'(0) &= 3/4 > 0 \\
f'(1.5) &= -3 < 0 \\
f'(2.5) &= -3 < 0 \\
f'(4) &= 8/9 > 0
\end{aligned}
$$

(e) A local max occurs at $x = c$ when $f(c)$ exists and $f'(x)$ changes from a positive (+) to a negative (−).

$$\text{Local Max: } (1, 2)$$

(f) A local min occurs at $x = c$ when $f(c)$ exists and $f'(x)$ changes from a negative (−) to a positive (+).

$$\text{Local Min: } (3, 6)$$

(g) Note: Even if $f'(x)$ changes signs at $x = 2$, it can't be a local max or min because $f(2)$ doesn't exist (it's a vertical asymptote).

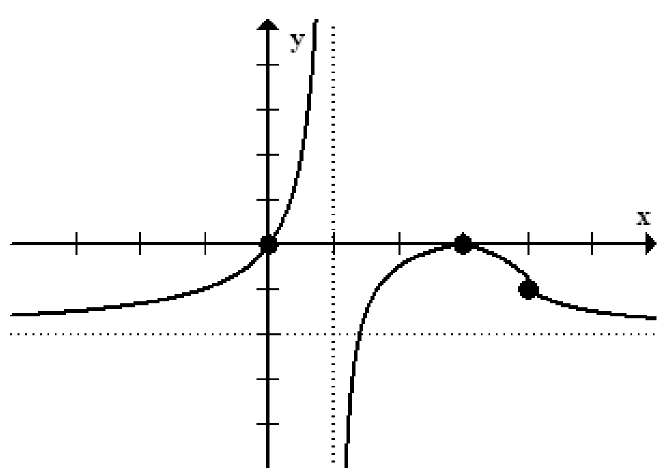
12. (6 points) Find the derivative $g'(x)$ of the function $g(x) = \int_5^{x^2} \sqrt{1+t}\, dt$

This is the Fundamental Theorem of Calculus Part I.

$$g'(x) = \sqrt{1+x^2} \cdot 2x$$

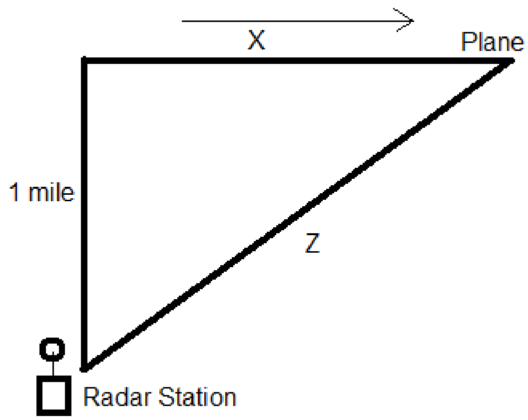
13. (10 points) Sketch the graph of a funcon satisfying all of the following conditions, labeling all asymptotes, local extrema, and inflection points.

(a) Vertical asymptote at $x = 1$
(b) Horizontal asymptote at $y = -2$
(c) x-intercepts (0,0) and (3,0)
(d) $f(4) = -1$
(e) $f'(x) > 0$ if $x < 3$, $x \neq 1$
(f) $f'(x) < 0$ if $x > 3$
(g) $f''(x) > 0$ if $x < 1$ or $x > 4$
(h) $f''(x) < 0$ if $1 < x < 4$



14. (12 points) A plane is flying horizontally at an altitude of 1 mile and a speed of 500 miles per hour directly over a radar station. Find the rate at which the distance from the plane to the radar station is increasing when the plane is 2 miles from the station.

(a) As always, draw a picture.



We are also given that $\frac{dx}{dt} = 500$.

We want to find $\frac{dz}{dt}$

(b) With related rates problems, we want to find an equation that relates all the variables together. In this case, it's Pythagorean's Theorem.

$$1^2 + x^2 = z^2$$

(c) Differentiate both sides

$$0 + 2x \cdot \frac{dx}{dt} = 2z \cdot \frac{dz}{dt}$$

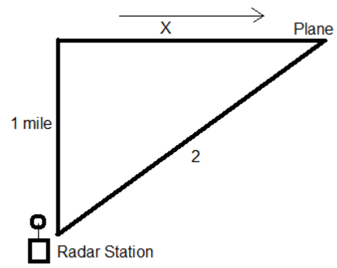
(d) Solve for $\frac{dz}{dt}$

$$2x \cdot \frac{dx}{dt} = 2z \cdot \frac{dz}{dt}$$

$$\frac{2x \cdot \frac{dx}{dt}}{2z} = \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt}}{z}$$

(e) We know $z = 2$. It's given in the problem. We also know $\frac{dx}{dt} = 500$. But we don't know x. Since we have a right triangle, we'll use Pythogorean's Theorem again.



$$1^2 + x^2 = 2^2$$
$$x^2 = 3$$
$$x = \sqrt{3}$$

(f) Now plug everything in to solve for $\frac{dz}{dt}$

$$\frac{dz}{dt} = \frac{\sqrt{3} \cdot 500}{2}$$

$$\frac{dz}{dt} = 250\sqrt{3}$$

(g) Answer: $\frac{dz}{dt} = 250\sqrt{3}$ miles per hour.

15. (12 points) Find two numbers x and y whose difference is 2500 and whose product is a minimum.

(a) No matter what optimization word problem you get, you need to find two equations. One is your objective and the other relates your variables together.

(b) If x and y are our two numbers, our objective is

$$\text{Minimize } P = xy$$

(c) We find minimums by taking the derivative. But I can't take the derivative of xy since I have two variables. We need another equation.

(d) The second equation:

$$x - y = 2500$$

(e) Now we can solve one variable for another

$$x = 2500 + y$$

(f) Plug $x = 2500 + y$ into the objective function

$$P = (2500 + y) \cdot y$$
$$P = 2500y + y^2$$

Our new objective is

$$\text{Minimize } P = 2500y + y^2$$

(g) Find $P'(y)$:

$$P'(y) = 2500 + 2y$$

(h) Solve $P'(y) = 0$ to find the critical values.

$$2500 + 2y = 0$$
$$2y = -2500$$
$$y = -1250$$

(i) To find x, plug $y = -1250$ into $x = 2500 + y$

$$x = 2500 + (-1250) = 1250$$

(j) Solution: Our two numbers with a difference of 2500 whose product is a minimum is $x = 1250$ and $y = -1250$.

16. (6 points) Approximate $\displaystyle\int_2^{10} \frac{1}{1+x}\,dx$ using the Riemann Sumn with $n = 4$ rectangles and right-hand endpoints.

(a) Graphing the function $f(x) = \dfrac{1}{1+x}$ might help. But if you can't, you'll need to use the following formulas.

(b) Find Δx

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{10-2}{4} = \frac{8}{4} = 2$$

(c) Using right-hand endpoints, we need to find x_1, x_2, x_3 and x_4.

$$x_i = a + i\Delta x$$

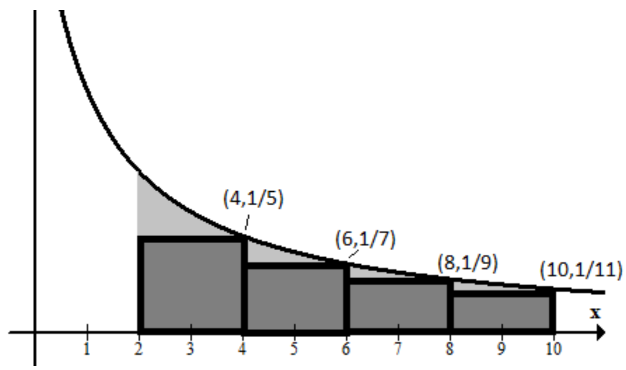
$$x_1 = 2 + (1)(2) = 4$$
$$x_2 = 2 + (2)(2) = 6$$

$$x_3 = 2 + (3)(2) = 8$$
$$x_4 = 2 + (4)(2) = 10$$

(d) Use the formula to find the area:

$$\begin{aligned} A &\approx \Delta x\,[f(x_1) + f(x_2) + f(x_3) + f(x_4)] \\ &= 2\,[f(4) + f(6) + f(8) + f(10)] \\ &= 2\left[\frac{1}{1+4} + \frac{1}{1+6} + \frac{1}{1+8} + \frac{1}{1+10}\right] \\ &= 2\left[\frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11}\right] \\ &= 1.08975 \end{aligned}$$

(e) If you are fortunate enough to be able to graph it, it will look like this:



Find the area of each rectangle. If you do it, you'll get 1.08975.

17. (28 points) Compute the following integrals.

(a) $\displaystyle\int \frac{x^6 - x^2}{x^4}\,dx$

i. Simplify the integrand.

$$\int \frac{x^6 - x^2}{x^4}\,dx = \int \frac{x^6}{x^4} - \frac{x^2}{x^4}\,dx = \int x^2 - x^{-2}\,dx$$

ii. Find the anti-derivative

$$\begin{aligned} \int x^2 - x^{-2}\,dx &= \int \frac{x^3}{3} - \frac{x^{-1}}{-1} + C \\ &= \frac{1}{3}x^3 + x^{-1} + C \end{aligned}$$

(b) $\displaystyle\int_0^{\pi/4} \frac{2\tan(x) - 3\sec^3(x)}{\sec(x)}\,dx$

i. Simplify the integrand.

$$\int_0^{\pi/4} \frac{2\tan(x) - 3\sec^3(x)}{\sec(x)}\,dx = \int_0^{\pi/4} \frac{2\tan(x)}{\sec(x)} - \frac{3\sec^3(x)}{\sec(x)}\,dx$$
$$= \int_0^{\pi/4} 2\sin(x) - 3\sec^2(x)\,dx$$

ii. Find the anti-derivative

$$\int_0^{\pi/4} 2\sin(x) - 3\sec^2(x)\,dx = -2\cos(x) - 3\tan(x)|_0^{\pi/4}$$

$$= [-2\cos(\pi/4) - 3\tan(\pi/4)] - [-2\cos(0) - 3\tan(0)]$$
$$= \left[-\sqrt{2} - 3\right] - [-2 - 0]$$
$$= -\sqrt{2} - 1$$

(c) $\int \cos(x) \cdot \sin(\sin(x))\,dx$

i. Let $u = \sin(x)$ and $du = \cos(x)\,dx$

ii. Solve for dx:

$$dx = \frac{du}{\cos(x)}$$

iii. Substitute

$$\begin{aligned} \int \cos(x) \cdot \sin(\sin(x))\,dx &= \int \cos(x)\sin(u) \cdot \frac{du}{\cos(x)} \\ &= \int \sin(u)\,du \\ &= -\cos(u) + C \\ &= -\cos(\sin(x)) + C \end{aligned}$$

(d) $\int_{-1}^{2} |x|\,dx$

Just like derivatives, you cannot take the anti-derivative of $|x|$. Recall that

$$|x| = \begin{cases} -x, & x \le 0 \\ x, & x > 0 \end{cases}$$

We use the property of integrals to split our integral so we can use x and $-x$.

$$\int_{-1}^{2} |x|\,dx = \int_{-1}^{0} -x\,dx + \int_0^2 x\,dx$$

i. $\int_{-1}^{0} -x \, dx$

$$\int_{-1}^{0} -x \, dx = -\frac{1}{2}x^2 \Big|_{-1}^{0}$$

$$= -\frac{1}{2}(0)^2 - \left(-\frac{1}{2}(-1)^2\right) = \frac{1}{2}$$

ii. $\int_{0}^{2} x \, dx$

$$\int_{0}^{2} x \, dx = \frac{1}{2}x^2 \Big|_{0}^{2}$$

$$\frac{1}{2}(2)^2 - \frac{1}{2}(0)^2 = 2$$

iii. Add the integrals together

$$\int_{-1}^{2} |x| \, dx = \frac{1}{2} + 2 = \frac{5}{2}$$