MATH 229, Exam III **BAILEY** NAME______________________________

1. (9 points) Carefully state the following theorems, making sure you have the hypotheses correct.
   - The Extreme Value Theorem
   - The Mean Value Theorem
   - Fermat's Theorem

2. (6 points) From which of the three theorems above can it be argued that if a drain pipe fills a 10 gallon bucket in 3 minutes, then at some point during that period the drain was flowing at rate of over 150 gallons per hour. Explain.

3. (15 points)

   (a) If Newton's method, with initial value $x_1 = 4$, is used to approximate $\sqrt{8}$, the positive zero of $f(x) = x^2 - 8$, what are x_2 and x_3?

   (b) Sketch the graph of $f(x) = x^2 - 8$. Show on your graph how Newton's Method constructs x_2 from x_1.

4. (10 points) Show that the equation $3 - 5x^3 - 6x^5 = 0$ has exactly one solution. State any theorems you use.

5. (15 points) Find the following limits. Justify your conclusions.

(a) $\displaystyle \lim_{x\to\infty} \frac{x^{3/2} - 2x^2 + 1}{3x^2 - 5x^3}$

(b) $\displaystyle \lim_{x\to\infty} (3x+1)\sin(\frac{1}{x})$

(c) $\displaystyle \lim_{x\to\infty} (x - \sqrt{x^2 + 3x + 4})$

6. (10 points) Let $f(x) = 11x + \frac{22}{x} - 10$.

   (a) Name the theorem that guarantees that f has both an absolute maximum and an absolute minimum on the interval $[-4, -1]$.

   (b) Find the absolute maximum and absolute minimum of f on $[-4, -1]$.

7. (15 points) Use calculus to find the point on the curve $y = \sqrt{x}$ that is closest to the point $(0, 108)$.

8. (20 points) Let $f(x) = \dfrac{x}{x^3 - 1}$.

(a) Find any vertical and horizontal asymptotes of the graph of f.

(b) Find the intervals of increase and decrease of f and all points $(a, f(x))$ for which $f(a)$ is a local maximum or a local minimum.

(c) Find the intervals on which f is concave upward and those on which it is concave downward. Find all inflection points $(b, f(b))$ of f.

(d) Sketch a good graph of f that plots all intercepts, local extrema, inflection points and vertical and horizontal asymptotes, and is consistent with all your answers above.