MATH 229, Exam I(A) NAME_____

1. (**15 points**) Let $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ \tan(x) & \text{if } 0 \leq x < \pi/2 \\ \frac{1}{x-\pi/2} & \text{if } x > \pi/2 \end{cases}$

   (a) Graph f.

   (b) Determine whether f is continuous at 0. Fully justify your conclusion.

   (c) Determine $\lim_{x \to \frac{\pi}{2}} f(x)$. Explain your answer.

2. (**10 points**) Sketch the graph of a single function f that satisfies all of the following conditions:

   - f has domain $[0, 4]$.
   - f is continuous on $(0, 2)$ and on $(2, 4)$.
   - $\lim_{x \to 2^-} f(x) = \infty$.
   - $\lim_{x \to 2^+} f(x) = 3$.

3. (**20 points**) Let $f(x) = 3x^{-2} + 5$.

   (a) Find a formula for $f'(x)$.

   (b) Find an equation for the line tangent to $y = f(x)$ at $x = 6$.

4. (**20 points**) Suppose that f, g and h satisfy

$f(3) = 7$	$g(3) = 5$	$h(3) = 7$
$\lim_{x\to 3} f(x) = 2$	g is continuous at 3	$\lim_{x\to 3} h(x) = 0^-$

Find the following quantities.

(a) $\lim_{x\to 3} \dfrac{1}{h(x)}$

(b) $\lim_{x\to 3} \cos(h(x))$

(c) $\lim_{x\to 3} (x^2+4)\sqrt{g(x)}$

(d) $\lim_{x\to 3} (5f(x)+1)^{-2}$

5. (**20 points**) Compute the following limits or explain why they do not exist. If the limit is infinite, you should state so.

(a) $\lim_{x \to 7} \dfrac{\sqrt{x+2} - 3}{x^2 - 49}$

(b) $\lim_{x \to \frac{\pi}{2}^+} \sec(x)$

(c) $\lim_{x \to 0} x^2 \cos(1/x)$ (Hint: Use the Squeeze Theorem.)

(d) $\lim_{x \to 6} \dfrac{3x - 18}{|x - 6|}$

6. (**5 points**) Graph $f(x) = \dfrac{\sin(x)\cos(x)}{\cos(x)}$ on the interval $[0, 2\pi]$.

7. (**10 points**)

   (a) State the Intermediate Value Theorem.

   (b) Show that the equation $\cos x = x$ has a root in the interval $[0, \pi]$.