Quiz 4 Math 211, Business Calculus, Spring 2014 Name:

1. Find $f'(x)$ when $f(x) = (x^2 - 4x^3)^5$

   Use the Chain Rule

$$f'(x) = 5(x^2 - 4x^3)^4 \cdot (2x - 12x^2)$$

   You do not need to simplify this one. Just know that it can be done.

2. Find $\frac{dy}{dx}$ when $y = (x+5)^2(x+4)^3$.

   This is a product rule between $(x+5)^2$ and $(x+4)^3$

$$f(x) = (x+5)^2 \qquad\qquad g(x) = (x+4)^3$$
$$f'(x) = 2(x+5) \qquad\qquad g'(x) = 3(x+4)^2$$

$$\begin{aligned}\frac{dy}{dx} &= f(x) \cdot g'(x) + g(x) \cdot f'(x) \\ &= (x+5)^2 \cdot 3(x+4)^2 + (x+4)^3 \cdot 2(x+5) \\ &= (x+5)(x+4)^2 \cdot [3(x+5) + 2(x+4)] \\ &= (x+5)(x+4)^2 \cdot [3x + 15 + 2x + 8] \\ &= (x+5)(x+4)^2(5x+23)\end{aligned}$$

3. Find y' and y'' when $y = 5x^2 + \frac{1}{x^2}$

   First, rewrite y as $y = 5x^2 + x^{-2}$.

$$y' = 10x - 2x^{-3} = 10x - \frac{2}{x^3}$$

$$y'' = 10 + 6x^{-4} = 10 + \frac{6}{x^4}$$

4. Find $\frac{d^2y}{dx^2}$ when $y = \frac{2x}{x+1}$.

It's best to start with the quotient rule.

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+1)\cdot 2 - 2x\cdot(1)}{(x+1)^2} \\ &= \frac{2x+2-2x}{(x+1)^2} \\ &= \frac{2}{(x+1)^2} \end{aligned}$$

At this point, you can continue using the quotient rule or you can rewrite $\frac{dy}{dx}$ as $\frac{dy}{dx} = 2(x+1)^{-2}$. I'll show you both ways.

Using $\frac{dy}{dx} = \frac{2}{(x+1)^2}$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{(x+1)^2\cdot 0 - 2\cdot 2(x+1)^1\cdot(1)}{(x+1)^4} \\ &= \frac{-4(x+1)}{(x+1)^4} \\ &= \frac{-4}{(x+1)^3} \end{aligned}$$

Using $\frac{dy}{dx} = 2(x+1)^{-2}$

$$\begin{aligned} \frac{d^2y}{dx^2} &= 2\cdot -2(x+1)^{-3}\cdot(1) \\ &= -4(x+1)^{-3} \\ &= \frac{-4}{(x+1)^3} \end{aligned}$$