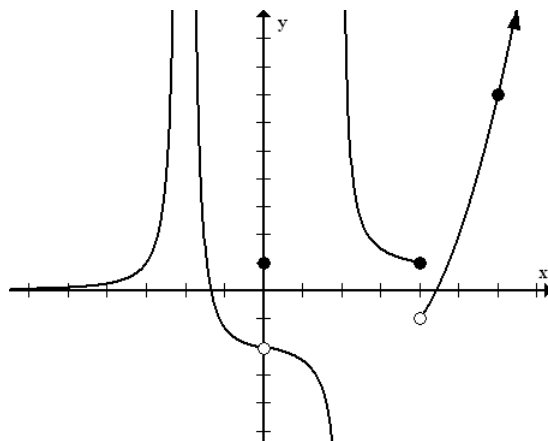


1. Consider the following graph. If a limit does not exist, write "DNE".



$$\lim_{x \rightarrow -2} f(x) = \infty$$

$$f(-2) = DNE$$

$$\lim_{x \rightarrow 0} f(x) = -2$$

$$f(0) = 1$$

$$\lim_{x \rightarrow 4^-} f(x) = 1$$

$$\lim_{x \rightarrow 4^+} f(x) = -1$$

$$\lim_{x \rightarrow 4} f(x) = DNE$$

$$\lim_{x \rightarrow 6} f(x) = 7$$

$$\lim_{x \rightarrow 2} f(x) = DNE$$

2. Is $f(x)$ continuous at $x = 0$. Explain.

(a) We check the three conditions for continuity.

(b) Does $f(0)$ exist?

Yes, $f(0) = 1$

(c) Does $\lim_{x \rightarrow 0} f(x)$ exist?

Yes, $\lim_{x \rightarrow 0} f(x) = -2$

(d) Does $\lim_{x \rightarrow 0} f(x) = f(0)$?

No, since $1 \neq -2$

(e) Since $\lim_{x \rightarrow 0} f(x) \neq f(0)$, $f(x)$ is not continuous at $x = 0$.

3. If $f(x) = \begin{cases} 8 - 2x & \text{if } x \leq 4 \\ \sqrt{x - 4} & \text{if } x > 4 \end{cases}$

Is $f(x)$ continuous at $x = 4$? Explain.

(a) We check the three conditions for continuity.

(b) Does $f(4)$ exist?

$$\text{Yes, } f(4) = 8 - 2(4) = 0$$

(c) Does $\lim_{x \rightarrow 4} f(x)$ exist? We need to check the left and right hand limits

$$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} 8 - 2x = 8 - 2(4) = 0$$

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} \sqrt{x - 4} = \sqrt{4 - 4} = 0$$

$$\text{therefore, } \lim_{x \rightarrow 4} f(x) = 0$$

(d) Does $\lim_{x \rightarrow 4} f(x) = f(4)$?

$$\text{Yes, } \lim_{x \rightarrow 4} f(x) = f(4) = 0$$

(e) Since $\lim_{x \rightarrow 4} f(x) = f(4)$, $f(x)$ is continuous at $x = 4$.

(f) I've supplied a graph of $f(x)$ as a way to verify the work shown above.

