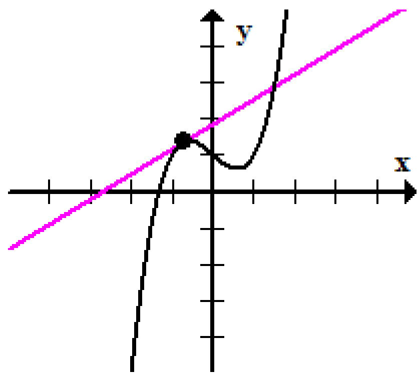
1.6 Tangents and Velocity Problems

Tangent Line

Suppose you have a graph of a function. And suppose you want to find a line that touches the graph at a certain point so that the slope of the line is the same as the slope of the graph at that point. It would look something like this:



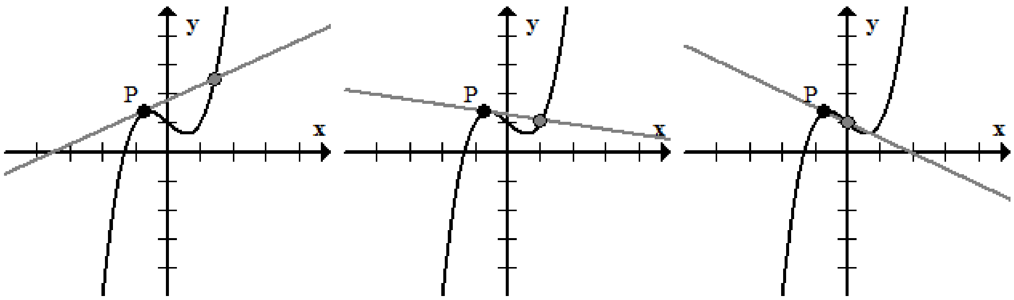
So how can we do this? It comes down to some algebra and the slope formula of a line. We'll come back to this idea in a bit.

Velocity Problems

We know how to find average velocity. Change in distance over change in time. In calculus, we want to go a bit deeper into the idea of velocity. Suppose you're driving on the highway and you look at your speedometer. It says you're going 55 mph. This is the speed you are traveling at that very moment. But how do you calculate it? Since at that very moment the change in time is 0, how do we actually calculate velocity at any given moment (called instantaneous velocity)?

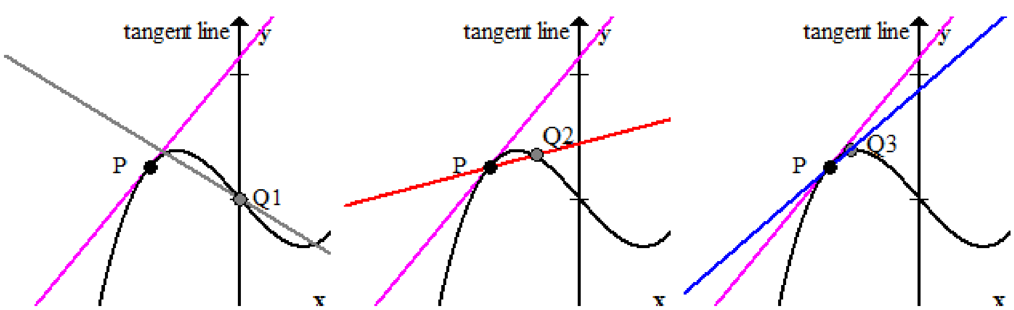
The Answer

To find the slope of a tangent (and the slope of a graph at a given point P), we find the slopes of many secant lines. What's a secant line you ask? I'll show you.



A secant line is simply a line connecting two points on a graph. So how does that help us find the tangent line? Here's the idea:

1. Make a secant line with the point P and some other point on the graph (call it $Q1$) and find the slope.
2. Pick a point closer to P. Call this $Q2$. Find the slope of this line.
3. Pick a point even closer to P. Call it $Q3$. Find the slope.
4. Continue this until your second point is extremely close to P.
5. The slope of these secant lines should be very close to the slope of the graph at P.



Can you see when the point Q gets closer to P, the secant lines are getting closer to looking like the tangent line? Since you know how to find the slope of a line, you then should be able to estimate the slope of the tangent line by using all the secant lines.

Instantaneous Velocity

The idea behind finding instantaneous velocity is very similar. Change in distance over change in time. Sound familiar? Because that's the slope formula (change in y over change in x). Let's take a look at an example.

Example 1.10.

Suppose you want to estimate the instantaneous velocity of a car after $t = 1$ seconds. For the sake of simplicity, the distance traveled is modeled after the following function

$$s(t) = 10t - 1.86t^2$$

You start by finding the average velocity during the following time intervals.

1. $t = 1$ to $t = 2$ seconds

$$\text{Average Velocity} = \frac{s(2) - s(1)}{2 - 1} = \frac{12.56 - 8.14}{1} = 4.42\ m/s$$

2. $t = 1$ to $t = 1.5$ seconds

$$\text{Average Velocity} = \frac{s(1.5) - s(1)}{1.5 - 1} = \frac{10.815 - 8.14}{.5} = 5.35\ m/s$$

3. $t = 1$ to $t = 1.1$ seconds

$$\text{Average Velocity} = \frac{s(1.1) - s(1)}{1.1 - 1} = \frac{8.7494 - 8.14}{.1} = 6.094\ m/s$$

4. $t = 1$ to $t = 1.001$ seconds

$$\text{Average Velocity} = \frac{s(1.001) - s(1)}{1.001 - 1} = 6.27814\ m/s$$

5. $t = 1$ to $t = 1.0001$ seconds

$$\text{Average Velocity} = \frac{s(1.0001) - s(1)}{1.0001 - 1} = 6.279814\ m/s$$

It looks like the velocity at $t = 1$ seconds is really close to 6.279814 m/s. So what's the real instantaneous velocity at $t = 1$ seconds? It's 6.28 m/s. Pretty close?