1.5 Composition of Functions

Definition 1.3 (Composition of Functions)**.** For functions $f(x)$ and $g(x)$, the composite function $f \circ g$ is

$$f \circ g = f\left(g(x)\right)$$

People struggle with composition for many reasons. Let's go through a series of examples that will build up to the definition we just stated for the composition of functions.

Function	Value
$f(x)$	$2x^2 - \cos(x) + 4x - 3$
$f(2)$	$2(2)^2 - \cos(2) + 4(2) - 3$
$f(-6)$	$2(-6)^2 - \cos(-6) + 4(-6) - 3$
$f(a)$	$2(a)^2 - \cos(a) + 4(a) - 3$
$f(2x+1)$	$2(2x+1)^2 - \cos(2x+1) + 4(2x+1) - 3$
$f(g(x))$	$2(g(x))^2 - \cos(g(x)) + 4(g(x)) - 3$

All of these are composition of functions. The last two are more obvious. A composition of functions is just evaluating a function, like $f(2)$. The difference is now we evaluate a function with another function. So $f(2x+1)$ is a composition of functions. Remember, all you're doing is plugging one function into another, just like how you would evaluate any function.

Example 1.9.

1. Let $f(x) = 3x^2 - 4x + 1$ and $g(x) = 3x - 5$. Find

   (a) $f \circ g$:

   First, always rewrite $f \circ g$ as $f\left(g(x)\right)$.

So, $f \circ g = f(g(x)) = f(3x-5) = 3(3x-5)^2 - 4(3x-5) + 1$.

(b) $(f \circ g)(2)$. This notation is really just asking for $f(g(2))$.

$$f(g(2)) = f(1) = 3(1)^2 - 4(1) + 1 = 0$$

2. Let $f(x) = \dfrac{1}{x}$ and $g(x) = x + 1$. Find the domain of

(a) $f \circ g$:

Let's just take a look at what $f(g(x))$ is.

$$f(g(x)) = f(x+1) = \frac{1}{x+1}$$

There are two ways of doing this. One way is just to find $f \circ g$ (do NOT simplify it) and simply find it's domain. From above you can see $x \neq -1$. Another way is to do it in steps. First, we look at the domain of $g(x)$. Since $g(x) = x + 1$, we have no domain issues. Ok, so we can plug anything into $g(x)$, but what about $f(x)$. Notice that we can't plug 0 into $f(x)$. And what do I plug into $g(x)$ that will give me $g(x) = 0$?

(b) $g \circ f$:

$$g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{x} + 1$$

We can never have a denominator equal 0, so $x \neq 0$. So the domain is all reals except $x = 0$.