

**Theorem 1.1**

*Let  $f$  be a non-negative, continuous function over an interval  $[a, b]$ . Let  $A(x)$  be the area between the graph of  $f$  and the  $x$ -axis over the interval  $[0, x]$ , with  $a < x \leq b$ . Then  $A(x)$  is a differentiable function of  $x$  and  $A'(x) = f(x)$ .*

Picture

**Example 1.2**

Find the area under the graph of  $f(x) = \frac{1}{5}x^2 + 3$  over the interval  $[2, 5]$ .

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**Definition 1.3**

Let  $f$  be any continuous function on  $[a, b]$  and  $F$  be any antiderivative of  $f$ . Then the **DEFINITE INTEGRAL** of  $f$  is

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**Example 1.4**

Evaluate the following definite integrals.

$$1 \quad \int_0^4 (5x + 3) dx$$

$$2 \quad \int_0^1 (1 - e^x) dx$$

$$3 \quad \int_1^e \left(x - \frac{1}{x}\right) dx$$

$$4 \quad \int_1^9 \frac{1+x}{\sqrt{x}} dx$$

$$5 \quad \int_1^2 \frac{2x + e^x}{xe^x} dx$$



**Theorem 1.5 (The Fundamental Theorem of Calculus)**

*If a continuous function  $f$  has an antiderivative  $F$  over  $[a, b]$ , then*

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**Example 1.6**

Find the area for the given graph  $y = x^2$  from  $x = -2$  to  $x = 0$ .

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**Example 1.7**

Find the area for the given graph  $y = \sqrt{x}$  from  $x = 1$  to  $x = 4$ .

**Example 1.8**

Find the area for the given graph  $y = \frac{1}{x}$  from  $x = 1$  to  $x = 7$ .

# Net Area

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## Example 1.9

Evaluate and interpret the results in terms of area:  $\int_0^2 (x^2 - x) dx$

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**Example 1.10**

Suppose that the marginal cost of making  $x$  hair clippers is  $C'(x) = 30 - \sqrt[3]{x}$  and the fixed cost is \$20,000. Find the cost of making 1,000 hair clippers.

You order an additional 500 hair clippers. How much more does it cost from the increase in hair clippers?



**Example 1.11**

A car accelerates at a constant rate from 0 mph to 60 mph in 30 seconds.

- 1 How fast is it traveling after 30 seconds?
- 2 How far has it traveled after 30 seconds?