Antiderivatives as Areas

Example 1.1

Find the area under the curve $f(x) = 4 - x$ over the interval [1,3].

Example 1.2

A company determines that for the first 50 crazy straws, its cost is \$5 per straw. What is the total cost to produce 30 straws?

Example 1.3

A fabrics firm has found that the cost per yard of producing x yards of a particular fabric is given by $c(x) = -0.007x + 12$, for $x \leq 350$, where $c(x)$ is the cost in dollars. Find the total cost of producing 200 yards of this material.

Example 1.4

A fruit juice producer has found that the marginal cost of producing x pints of orange juice is $C'(x) = 0.000008x^2 - 0.004x + 2$, for $x \leq 350$ where $C'(x)$ is in dollars per pint. Approximate the total cost of manufacturing 270 pints of juice, using 3 subintervals and the left endpoint of each subinterval.

Example 1.5

Consider the function $f(x) = 4 - x^2$ over the interval $[-1, 1]$.

1. Approximate the area bounded by the curve and the x-axis over $[-1, 1]$ by dividing the interval into two subintervals.
2. Approximate the area bounded by the curve and the x-axis over $[-1, 1]$ by dividing the interval into four subintervals.
3. Approximate the area bounded by the curve and the x-axis over $[-1, 1]$ by dividing the interval into eight subintervals.
4. How can I find the exact area over the interval $[-1, 1]$?

Definition 1.6

Let $f(x)$ be a continuous function with $f(x) \geq 0$ over $[a, b]$. A **definite integral** is the limit as $n \to \infty$ (or, $\Delta x \to \infty$) of the area of the rectangles under the graph of $y = f(x)$ over $[a, b]$.

Exact Area $A =$